GRAMMAR SCHOOL ALGEBRA

MILNE

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A COURSE FOR GRAMMAR SCHOOLS AND BEGINNERS IN PUBLIC AND PRIVATE SCHOOLS

BY

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PREFACE.

THIS book is designed to present the elementary facts of the science of algebra in such a manner that a deep interest will be awakened in the processes.

The author has in several instances departed from the order of classification commonly followed in text-books on this subject because he deems it better to introduce the student to the attractive features of the science at an early stage rather than to wait until he has mastered all the processes employed with most forms of algebraic quantities.

The ideas of number which the pupil has gained in arithmetic have been associated with those involved in algebra in such a way that no difficulty is experienced in passing from reasoning upon definite numbers to reasoning upon general numbers.

The treatment of equations is introduced at the beginning, and it is presented throughout the book, wherever it is possible to do so advantageously.

The method of presentation exemplified in the other books of the series has been followed here, because it is recognized as pedagogically correct and because it has met with general approval.

The work presents the merest elements of the science, and yet it is believed that the method of presentation, the illustration and application of mathematical principles, and the knowledge gained from the solution of problems, will familiarize the student with the fundamental principles of the science to such a degree that easy and rapid progress in the more abstract phases of the subject will be secured whenever he pursues the subject farther.

The arrangement is such that those who prefer to omit the sections upon Simultaneous Equations and Equations solved by Factoring may do so without encountering any difficulty in the subsequent work.

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ALGEBRAIC PROCESSES.

1. PROBLEM. A farmer had 444 sheep in two fields, one of which contained three times as many sheep as the other. How many sheep had he in each field?

ARITHMETICAL SOLUTION.

	A certain number = the number in one field.
Then	3 times that number = the number in the other field.
And	4 times that number = the number in both fields.
Therefore	4 times that number = 444.
	The number = 111, the number in one field.
	3 times 111 = 333, the number in the other field.

The solution given above may be abbreviated by using the letter n for the expressions *a certain number* and *that number*. In algebra, however, it is customary to represent a number whose value is to be found by x or by some other one of the last letters of the alphabet.

ALGEBRAIC SOLUTION.

Let	$x =$ the number in one field.
Then	$3x =$ the number in the other field.
And	$4x =$ the number in both fields.
Therefore	$4x = 444$.
	$x = 111$, the number in one field.
	$3x = 333$, the number in the other field.

2. The student will observe that algebra is very much like arithmetic, but that it differs from arithmetic in often using letters to represent numbers.

Letters used to represent numbers are usually called *quantities,* though it is also proper to call them numbers.

Thus, x in the preceding algebraic solution is called a quantity.

3. Letters used to represent numbers or quantities whose values are to be found are called **Unknown Numbers or Quantities.**

The *last letters* of the alphabet, as x, y, z, etc., are commonly employed to represent *unknown numbers or quantities.*

Thus, x in the solution already given is called an unknown quantity.

4. An expression of equality between two numbers or quantities is called an **Equation.**

Thus, $5 + 8 = 13$, and $3x = 15$ are called equations.

5. The sign $=$ is used to indicate equality.

6. The sign $\therefore$ is used, in writing solutions of problems, instead of the word *therefore* or *hence.* It is called the sign of **Deduction.**

PROBLEMS.

7. 1. John and James together had 40 marbles, but John had three times as many as James. How many had each?

2. Two boys together dug 60 bushels of potatoes. If one dug twice as many bushels as the other, how many did each dig?

3. If A and B together have \$3000, and A has twice as much as B, how much has each of them?

4. A number added to five times itself equals 30. What is the number?

5. If a man is three times as old as his son, and the sum of their ages is 40 years, how old is the father? How old is the son?

6. Divide the number 75 into two parts such that one part may be four times the other.

7. A man bought a horse and carriage for $600, and the carriage cost him twice as much as the horse. What did each cost him?

8. A certain orchard has 700 trees in it. There are twice as many cherry trees as pear trees, and twice as many apple trees as cherry trees. How many are there of each kind?

9. A, B, and C were talking of their ages. A said, "I am twice as old as B"; B said, "I am twice as old as C." The sum of their ages was 140 years. How old was each?

SUGGESTION. Discover from the problem the number which, if known, will enable you to find all the required numbers, and let x stand for that number. In this problem x will stand for C's age.

10. The greater of two numbers is six times the less, and their sum is 49. What are the numbers?

11. After taking five times a number from fifteen times the same number, and adding to the remainder six times the number, the result was 8 more than 72. What was the number?

12. Mary has five times as many books as Hannah, Hannah has two times as many as Jane, and together they have 39 books. How many has each?

13. I paid five times as much for pens as for ink, and three times as much for paper as for both of these. How much did I pay for each, if they all cost me 96 cents?

14. A man paid twice as much for a pair of trousers and seven times as much for an overcoat as for a hat. How much did each cost him, if they all cost $60?

15. A man divided $50 among four boys. To the second he gave twice, to the third three times, and to the fourth four times, as much as to the first. How much did he give to each?

16. A farmer has four flocks of sheep. The second is twice as large as the first, the third three times as large, and the fourth as large as the first and third together. If he has 200 sheep in all, how many are there in each flock?

17. If one number is six times another number, and a third number is 10 times that number, and the sum of the three is 119, what are the numbers?

18. A man failed in business, owing A ten times as much as B, C four times as much as B, and D eight times as much as the difference between what he owed A and C. The entire amount he owed these men was $6300. How much did he owe each?

19. If a gold watch is worth ten times as much as a silver watch, and both together are worth $132, how much is each watch worth?

20. The whole number of votes cast for A and B at a certain election was 450, but four times as many votes were cast for A as for B. How many were cast for each?

21. A man paid $175 for a horse, a cow, and a wagon. He paid twice as much for the horse as for the wagon, and

twice as much for the wagon as for the cow. How much did he pay for each?

22. A, B, and C form a partnership with a capital of \$14,000. B and C each furnish three times as much as A. How much capital does each furnish?

23. A and B performed a certain piece of work for which B received three times as much as A, and A received \$12 less than B. How much did each receive?

SOLUTION. Let $x =$ the amount A received.

Then $3x =$ the amount B received.

$2x =$ the amount B received more than A.

$\therefore\ 2x = \$12.$

$x = \$6$, the amount A received.

$3x = \$18$, the amount B received.

24. A farmer raised five times as many acres of wheat as of oats, and five times as many acres of oats as of potatoes. If 186 acres were cultivated, how many acres were there of each?

25. A man's capital doubled for three successive years. If he then had \$9600, how much had he at first?

SUGGESTION. Let $x =$ his original capital.

Then $2x =$ his capital at end of first year.

26. Sarah had seven times as many chickens as ducks, and twice as many ducks as hens. If there were in all 119 fowls, how many were there of each kind?

27. A certain number can be separated into two factors, one of which is three times the other, and whose sum is 36. What are the two factors?

28. One cask holds four times as much as another, and the larger contains 27 gallons more than the smaller. How many gallons are there in each cask?

29. The population of one town is eight times the population of another, and the larger one contains 4900 more inhabitants than the smaller. What is the population of each town?

30. The difference between a certain number and that number multiplied by 18, is 119. What is the number?

31. A, B, and C work the same number of days. A receives 75 cents per day, B receives 90 cents, and C receives 95 cents. All together receive $13. How many days does each work?

32. A father's age is five times the age of his son, and the difference between their ages is 44 years. What is the age of each?

33. What number added to $\frac{1}{2}$ of itself equals 12?

SOLUTION. Let $x =$ the number.

Then $x + \frac{1}{2}x = 12$

Or $\frac{3}{2}x = 12$

$\therefore \frac{1}{2}x = 4$

And $x = 8$

Hence the number is 8.

34. If a certain number is added to $\frac{1}{3}$ of itself, the sum is 20. What is the number?

35. If to six times a certain number $\frac{1}{3}$ of itself is added, the sum is 19. What is the number?

36. A farmer had $\frac{3}{7}$ as many horses as cows, and altogether he had 10 cows and horses. How many of each had he?

37. One of my pear trees bore $\frac{2}{7}$ as many bushels of pears as the other, and both together bore 18 bushels. How many did each tree bear?

38. There is a certain number, $\frac{1}{4}$ of which exceeds $\frac{1}{6}$ of it by 2. What is the number?

39. One half of a certain number exceeds $\frac{1}{3}$ of it by 3. What is the number?

40. Divide 45 into two parts such that one may be $\frac{2}{3}$ of the other.

41. One half and one third of a certain number added together make 10. What is the number?

42. Three fourths of a number added to two fifths of it makes 23. What is the number?

43. If Charles had five times as many marbles as he now has, he would have as many as John and William together, the former of whom has 30, and the latter 15. How many marbles has Charles?

44. If a fourth of a number is subtracted from $\frac{1}{2}$ the number, the result is 3. What is the number?

45. A man had 260 sheep in three fields. In the second he had twice as many as in the first, and in the third $\frac{1}{4}$ of the number in the first. How many had he in each field?

46. The less of two numbers is $\frac{1}{5}$ of the greater, and their sum is 14 less than 50. What are the numbers?

47. Divide 120 into three such parts that the first part is $\frac{1}{2}$ of the second, and $\frac{1}{3}$ of the third.

SUGGESTION. Let $x =$ the first part.
Then $2x =$ the second part.

48. A, B, and C together have $900. A has twice as much as B, and C has twice as much as A and B together. How much has each?

49. What number is that to which if you add its half and take away its third, the remainder will be 147?

50. James has $\frac{1}{3}$ as many marbles as Henry, and Henry has 16 more than James. How many marbles has each?

51. Julia has $\frac{1}{3}$ as much money as May; Hattie has $\frac{1}{2}$ as much as Julia; all together have 18 cents. How many cents has each?

52. A certain number plus $\frac{1}{3}$ of itself, plus $\frac{1}{5}$ of itself equals 46. What is the number?

53. In Mr. Green's library there were 1900 volumes. The number of books of fiction was $\frac{2}{5}$ the number on science; the number on science was twice the number on history. How many volumes were there of each?

54. What number increased by $\frac{9}{10}$ of itself equals 133?

55. B has $\frac{3}{5}$ as much money as A; C has $\frac{1}{3}$ as much as B; altogether they have \$27. How much has each?

56. Two fifths of a number plus $\frac{1}{2}$ of the number equals 36. What is the number?

57. George sold a certain number of oranges at 2 cents apiece; Walter sold $\frac{1}{3}$ as many at 4 cents apiece. If both together received 90 cents, how many oranges did each sell?

SOLUTION. Let $x =$ the number George sold.

Then $\frac{1}{3}x =$ the number Walter sold.

$2x =$ the number of cents George received.

$\frac{4}{3}x =$ the number of cents Walter received.

$\frac{4}{3}x + 2x =$ the number of cents both received.

$\therefore \frac{10}{3}x = 90.$

$\frac{1}{3}x = 9.$

$x = 27$, the number of oranges George sold.

$\frac{1}{3}x = 9$, the number of oranges Walter sold.

58. Robert bought a certain number of apples, and his mother gave him $\frac{4}{5}$ as many as he bought. He then had 18. How many apples did he buy?

59. A man gave \$50 to three poor families. To the first he gave $\frac{1}{2}$ as much as to the second, to the third seven times as much as to the first. How much did he give to each?

60. Three newsboys sold 100 papers. The first and the second sold the same number, while the third sold $\frac{2}{3}$ as many as the other two. How many did each sell?

61. A and B start in business with the same amount of money. A gains $\frac{1}{2}$ as much as he had, while B loses $\frac{1}{5}$ of his capital; they then have together \$1150. What was the original capital of each?

62. What number increased by $\frac{1}{9}$ of $\frac{3}{4}$ of itself equals 52?

63. The length of a field is $1\frac{1}{2}$ times its breadth, and the entire distance around the field is 60 rods. What are the length and the breadth of the field?

SOLUTION.	Let $x =$ the number of rods in the breadth.
Then	$1\frac{1}{2}x =$ the number of rods in the length.
And	$2\frac{1}{2}x =$ half the distance around the field.
	$\therefore\ 5x = 60$ rods, the distance around the field.
	$x = 12$ rods, the breadth of the field.
	$1\frac{1}{2}x = 18$ rods, the length of the field.

64. What is the number whose double and half added together give 35?

65. The sum of three numbers is 34. The first multiplied by 4 produces the second, and $\frac{1}{2}$ of the first taken from the second gives the third. What are the numbers?

66. Find two factors of 100, one of which is four times the other, and whose sum is $\frac{1}{4}$ of 100.

67. A father gave to his first son three times as much as to the second, to the third $\frac{1}{2}$ as much as to the first and second; in all he gave \$600. How much did he give to each?

68. Four men are taxed in the aggregate \$500 as follows: for every dollar that the first pays the second pays 1 dollar, the third pays $\frac{1}{2}$ of a dollar, and the fourth pays $1\frac{1}{2}$ dollars. How much is each man's tax?

69. Emma sold 10 quarts of berries at a certain price per quart. Anna sold 14 quarts at twice that price per quart. Together they received \$1.90. What price per quart did each receive?

70. Two boys walked toward each other from two towns 18 miles apart. If one boy walked $\frac{4}{5}$ as far as the other, how many miles did each walk?

71. What number is that whose fourth part exceeds its fifth part by 3?

72. A man paid a certain sum for a house and lot, and sold them at a gain equal to $\frac{1}{2}$ the cost. If the selling price was \$3000, what was the cost?

SOLUTION. Let $x =$ the cost of the house and lot.

Then $\frac{1}{2}x =$ the gain.

And $1\frac{1}{2}x =$ the selling price.

$\therefore \frac{3}{2}x = 3000.$

$\frac{1}{2}x = 1000.$

$x = 2000$, the cost of the house and lot.

73. A man paid a debt of \$7500 in 4 months, paying each month twice as much as the month before. How much did he pay the first month?

74. A man traveled 324 miles; he went three times as far by steamboat as by stage, and eight times as far by railroad as by steamboat. How many miles did he travel by each conveyance?

75. Harry solved 8 problems more than Edward, and Edward solved $\frac{1}{3}$ as many as Harry. How many did each solve?

76. A man paid $150 for a wagon, which was 50 per cent or one half more than the original cost. What was the original cost of the wagon?

77. A man bought a lot for a certain sum; he built upon it a house costing twice as much as the lot; he furnished the house at an expense equal to $\frac{1}{2}$ the price of the lot. The man, who had only $6000 in cash, found that he could pay for all his expenditures except furnishing the house. What was the amount of each item?

78. In an orchard there are 210 trees, arranged in rows. There are 5 rows with a certain number of trees in a row, and 8 rows with twice that number of trees in a row. How many trees are there in the different rows?

79. A man earned daily for 4 days three times as much as he paid for his board. After paying his board for 4 days, he had $8 left. How much did he receive per day, and how much did he pay for his board?

80. A man on being asked how old he was said that $\frac{1}{10}$ of his age added to twice his age made 84 years. How old was he?

81. A man borrowed as much money as he had, and then spent $\frac{1}{3}$ of the whole sum. If he had $4 left, how much had he at first?

82. A man had twice as many 5-cent pieces as dimes, and his money amounted to 200 cents. How many pieces had he of each kind?

SOLUTION. Let $x =$ the number of dimes.

Then $2x =$ the number of 5-cent pieces.

And $10x =$ the number of cents in the dime pieces.

$10x =$ the number of cents in 5-cent pieces.

$\therefore 20x = 200.$

$x = 10$, the number of dimes.

$2x = 20$, the number of 5-cent pieces.

83. A sum of money amounting to \$5 is composed of half and quarter dollars. The number of half dollars is double the number of quarter dollars. How many pieces are there of each kind?

84. A man worked 10 days for a certain sum per day; his wife also worked at the same rate, and his son at $\frac{1}{5}$ the rate of his father. They received in all \$27.50. What were the daily wages of each?

85. In a certain field the length is twice the breadth, and the distance around the field is 90 rods. What are the length and the breadth of the field?

86. A tree 45 feet high was broken off so that the part left standing was twice the part broken off. What was the length of each part?

87. Divide \$800 among three men so that the first and third shall together have three times as much as the second, and the first shall have double what the third has.

88. The income from a very successful business quadrupled every year for three years. If the entire income for the three years was \$42000, what was the income for each year?

ALGEBRAIC EXPRESSIONS.

8. Quantities connected by algebraic signs are called **Algebraic Expressions.**

Thus, $a + b$ and $2a - 3bc$ are algebraic expressions.

9. The signs employed in arithmetic are generally used for the same purposes in algebra.

What do the following expressions indicate?

1. $2 + 5$; $17 - 6$; $16 \div 4$; 15×3.
2. $a + b$; $c - d$; $c \div d$; $d \times a$.
3. $a + b + c$; $a - b + c$; $a - b - c$; $a \times b \times a$.
4. $a - b - c + d$; $a + b - d - e$; $a \times b \times c \times d$.

Multiplication is also indicated by writing letters, or a letter and a figure, side by side without any sign between them, or with a dot between them. Thus, $a \times b \times c$ may be written abc or $a \cdot b \cdot c$.

What do the following expressions indicate?

5. $4ab$; $5ab$; $3a + 2b$; $4x + 3y$.
6. $4xyz$; $6abc$; $6ax + 3ay$; $5cd - 3d$.
7. $3xy - 4yz + 2z$; $4xyz - 2axy - 3by$.

10. The quantities which when multiplied together form a product are called the **Factors** of the product.

11. The product arising from using a quantity a certain number of times as a factor is called a **Power** of that quantity.

Thus, 4 is a power of 2; 64 is a power of 4 and also of 8.

Powers are indicated by a small figure or letter, called an **Exponent,** written a little above and at the right of the quantity, showing the number of times the quantity is to be used as a factor.

Thus, a^5 shows that a is to be used as a factor *five* times, or that it is equal to $a \times a \times a \times a \times a$.

Powers are named from the number of times a quantity is used as a factor.

Thus, a^7 is read the *seventh power* of *a* or *a seventh.*

The *second* power is also called the *square,* and the *third* power the *cube* of a quantity.

12. Read the following expressions and state what they indicate.

1. a^8; x^6; x^9; x^2y^2; x^3y^3; x^2y^3; x^5y^4.
2. a^2+b^2; a^3+b^3; a^5+b^5; a^8-b^8.
3. a^2b+ab^2; $a^3x-b^3y^2$; $a^2x^3+bx^3$; $a^4x^2y-b^5x^3y^2$.
4. $a^2+b^2-c^2$; $a^2x^2+b^2y^2+c^2z^2$; $axy^3+by^2z^2-y^3$.

13. A figure or letter placed before a quantity to show how many times it is taken additively is called a **Coefficient.**

Thus, in the expression $5x$, 5 indicates that x is to be taken five times, or that it is equal to $x+x+x+x+x$.

In the expression $4bc$, 4 may be regarded as the coefficient of bc, or $4b$ may be regarded as the coefficient of c.

When no coefficient is written, it is manifestly 1.

14. What do the following expressions indicate?

1. $5a^2x^3z^5$.

INTERPRETATION. The expression indicates five times the product of a used twice as a factor, multiplied by x used three times as a factor, multiplied by z used five times as a factor.

It is usually read *five a square, x cube, z fifth.*

2. $3x^2y^2$.	5. $2x^2+y^2$.	8. $3x^2+2y-3z$.
3. $4x^3y^3$.	6. $3x-4y^2$.	9. $2a^2x^2-3xy+2z^2$.
4. $5axy^4$.	7. $4xy-3y^3$.	10. $5a^2y+3x^2y^2+5cz^2$.

15. When several quantities are inclosed in parentheses, ( ), they are to be subjected to the same process.

Thus, $2 \times (x + y)$ indicates two times the sum of x and y; $3(a + b - c)$ indicates three times the remainder when c has been subtracted from the sum of a and b.

16. 1. Interpret and read the expression $a \times (b + c)$ or $a(b + c)$.

INTERPRETATION. The expression means that the sum of b and c is to be multiplied by a. It is usually read a times the *quantity b plus c.*

Interpret and read the following:

2. $x(y + z)$.
3. $5(a - c)$.
4. $3d(c - y)$.
5. $4x(c + 2d)$.
6. $3ac(2x - 3y^2)$.
7. $a + 4d(ax - cy)$.
8. $3x + 4(2y - 3z)$.
9. $2(3x - 5y) + (6x - 3y)$.
10. $3a(x + c) - 5y(z + d)$.
11. $5ax(a - b) - 3cy(c + d)$.

Write the following in algebraic expressions:

1. The sum of five times a and three times the square of x.

SOLUTION. $5a + 3x^2$.

2. Three times b, diminished by 5 times a raised to the fourth power.

3. The product of a, b, and $a - c$.

4. Seven times the product of x times y, increased by three times the cube of z.

5. Three times x, diminished by five times the sum of a, b, and c.

6. Six times the square of m, increased by the product of m and n.

7. The product of a used five times as a factor, multiplied by the sum of b and c.

8. Twelve times the square of a, diminished by five times the cube of b.

9. Eight times the product of a and b, diminished by four times the fourth power of c, or c used four times as a factor.

10. Six times the product of the second power of a multiplied by n, increased by five times the product of a times the second power of n.

11. Four times the product of x by the second power of y, multiplied by the cube of z, diminished by a times the fourth power of c.

12. The fourth power of a plus the cube of b, plus three times the product of the square of a by the square of b, diminished by the cube of d.

17. When $a=1$, $b=2$, $c=3$, $d=4$, $e=5$, find the numerical value of each of the following expressions by using the number for the letter which represents it.

Thus, $2a+3b-c^2+d=2+6-9+4=3$.

1. $3abcd$.
2. $5abcd^2$.
3. $2a^2b^2c^2$.
4. $3abcde$.
5. $3a+5b$.
6. $4a+3b$.
7. $2e-3c+d$.
8. $4ab^2cd^2$.
9. $10a^2b-3ab^2$.
10. $3d-2b^2+e$.
11. $5cd-5bc-5ab$.
12. $8a^2b^2+c^2-de$.
13. $12bc+e(c-a)$.
14. $3c^2d-5abc$.
15. $9a-4b+2cd$.
16. $3abd-4e+cd$.
17. $a^2b^2c^2-d^2+e^2$.
18. $(a+b)+e(d-c)$.
19. $b(d-a+c)-bd$.
20. $abc+ade-d(e-c)$.
21. $b^2c^2d^2-ab+cd$.
22. $b(d+a)+e(b+c)$.
23. $c^2+4abc+d(e-d)$.
24. $(a+b)+3(c+d)+2(e-d)$.

TERMS IN ALGEBRAIC EXPRESSIONS.

18. An algebraic expression whose parts are not separated by the signs + or − is called a **Term.**

Thus, $5x$, $3by$, $4x^2y^2z^2$, $a^2b^2d^2e$ are terms.

In the expression $3x + 2ay + 3cd - z$ there are *four* terms; in the expression $xy^2 - x^3y^2$ there are but *two* terms.

19. A term which has the sign + before it is called a **Positive Term.** When the first term of an expression is positive, the sign + is usually omitted before it.

Thus in the expression $2a - 3b + 4c - 5d + 3e$ the first, third, and fifth terms are *positive.*

20. A term which has the sign − before it is called a **Negative Term.**

Thus in the expression $2a - 3b + 4c - 5d - 3e$, the second, fourth, and fifth terms are *negative.*

21. Terms which contain the same letters with the same exponents are called **Similar Terms.**

The coefficients or signs need not be alike, however.

Thus, $5x^3$ and $-7x^3$ are similar terms; also $3(x+a)^5$ and $5(x+a)^5$.

Expressions like bx^3 and cx^3 may be considered similar terms by regarding b and c as coefficients.

22. Terms which contain different letters, or the same letters with different exponents, are called **Dissimilar Terms.**

Thus, $3xy$ and $3pq$ are dissimilar terms; also $5x^2y$ and $5xy^2$.

23. An algebraic expression consisting of but one term is called a **Monomial.**

Thus, ab, $3axyz$, $5x^2y^2z$ are monomials.

24. An algebraic expression consisting of more than one term is called a **Polynomial.**

Thus, $a + b + c$ and $3x + 4y$ are polynomials.

25. A polynomial of two terms is called a **Binomial.**

Thus, $3x + 2y$, $a - b$, and $4ab - 3cd$ are binomials.

26. A polynomial of three terms is called a **Trinomial.**

Thus, $a + b + c$ and $2x - 2y + 5z$ are trinomials.

27. Select from the following:

1st, the positive terms.
2d, the negative terms.
3d, the similar terms.
4th, the dissimilar terms.
5th, the monomials.
6th, the binomials.
7th, the trinomials.
8th, the polynomials.

1. $2ax$, $3x^2y$, $2a + 3b$, $6x^2y$, $5ax^2$, $3x + 2b$.

2. $3x^2 + 2y^2$, $5a - 3b + c$, $-4ax$, $6a - c + d$, $a^2 + b^2$

3. $a + b + 2c + 3d$, $ax - bx - 3c + 2d$, $ax + bx$.

4. $a^2x^2 + 2ax$, $3a + b$, $2a - c + d$, $3a^2x^2 - 4ax$.

Write the following:

5. Four positive terms; three negative terms.

6. An expression containing three positive and two negative terms.

7. Three similar terms; four dissimilar terms.

8. An expression containing four positive similar terms, and one containing four positive dissimilar terms.

9. An expression containing five similar terms, three of which are positive and two negative.

10. An expression containing five dissimilar terms, four of which are negative and one positive.

11. Six positive monomials; three positive similar monomials; four negative monomials; three negative similar monomials.

12. Four polynomials; four binomials; four trinomials.

POSITIVE AND NEGATIVE QUANTITIES.

28. In arithmetic the signs + and — are used to indicate operations to be performed, but in algebra they are used also as **Signs of Opposition.**

Thus if *gains* are considered *positive* quantities, *losses* will be *negative;* if distances *north* from a given point are considered *positive,* distances *south* will be considered *negative,* etc.

1. Place appropriately before each of the following quantities the sign + or the sign —:

Mr. A gains $40 and loses $20. Mr. S earns $25 and spends $15. A ship sails 40 miles north from a given parallel, and then 20 miles south. A thermometer indicates on Monday 15° above zero, and on Wednesday 10° below zero.

2. A man deposits in a bank $50, and then draws out $20. Indicate the transaction by using the signs + and —.

3. Mr. H buys 25 horses and sells 10 of them; he then buys 15 more and sells 8 of them. Indicate the transactions by proper signs.

4. A vessel sailed 40 miles east, and was then driven by adverse winds 30 miles west. Indicate the directions by proper signs.

5. How much worse off is a man who is $50 in debt than if he had nothing? Indicate his condition by the sign + or —.

A negative quantity is sometimes regarded as less than zero.

6. Two vessels left the same port, one sailing west 10 miles per hour and the other sailing east 8 miles per hour. How far were they apart at the end of two hours? Indicate the distance sailed by each vessel and the direction.

ADDITION.

29. 1. How many days are 5 days, 4 days, and 3 days?

2. How many d's are $5d$, $4d$, $3d$, and $6d$?

3. How many c's are $6c$, $3c$, $5c$, and $2c$?

4. How many ab's are $3ab$, $2ab$, $4ab$, and $5ab$?

5. When no sign is prefixed to a quantity, what sign is it assumed to have?

6. When positive quantities are added, what is the sign of the sum?

7. If Henry owes one boy 3 cents, another 5 cents, and another 6 cents, how much does he owe?

8. If the sign $-$ is placed before each sum that he owes, what sign should be placed before the entire sum?

9. What sign will the sum of negative quantities have?

10. If a vessel sails $+5$ mi., $+8$ mi., $+9$ mi., and then sails -4 mi., -2 mi., -6 mi., how far is she from the sailing port?

11. A boy's financial condition is represented as follows: Henry owes him 15 cents; David owes him 10 cents; James owes him 8 cents. He owes William 9 cents, and Fred 10 cents. What is his financial condition? How much is 15 cents, 10 cents, 8 cents, -9 cents, -10 cents?

30. PRINCIPLE. *Only similar quantities can be united by addition into one term.*

Dissimilar quantities cannot be added, but in algebra an *indicated* operation is often regarded as an operation performed. It is important to remember that fact.

Thus, a and b cannot be added, and yet $a + b$ is called their sum, though the operation is only indicated.

31. To add similar monomials.

1.	2.	3.	4.	5.	6.
$3x$	$9ab$	$-3mn$	$-x^2y$	ax	$-bc$
$7x$	$4ab$	$-2mn$	$-5x^2y$	$6ax$	$-7bc$
x	$3ab$	$-9mn$	$-8x^2y$	$5ax$	$-2bc$
$5x$	$6ab$	$-mn$	$-4x^2y$	$4ax$	$-12bc$
$2x$	$2ab$	$-5mn$	$-7x^2y$	$3ax$	$-6bc$

7. Express in the simplest form $5a + 3a - 2a - 7a + 12a + 3a$.

SOLUTION. The sum of the *positive* quantities is $23a$, and the sum of the *negative* quantities $-9a$. $23a - 9a = 14a$. Hence the sum of the quantities, or the simplest form of the expression, is $14a$.

Express in the simplest form:

8. $8a - 2a + a - 3a - a + 7a$.
9. $4x^2y^2 + 3x^2y^2 - x^2y^2 + 5x^2y^2 - 10x^2y^2$.
10. $7mx + 4mx - 5mx - 2mx - 6mx + mx$.
11. $5y - 3y + 8y - 10y + 6y - y$.
12. $8m + 3m - 5m - 2m + 6m - 4m$.
13. $7bc + 3bc - 4bc - 5bc + 8bc - bc$.
14. $9xy + 2xy - 5xy + 10xy - 7xy - 4xy + 5xy$.
15. $6x^2z - 4x^2z + 3x^2z + 8x^2z - 5x^2z + 3x^2z - 10x^2z$.

16. $15\,mn + 6\,mn - 10\,mn - 4\,mn - 3\,mn + 4\,mn.$

17. $4\,a^2b - 3\,a^2b + 7\,a^2b - 14\,a^2b - 3\,a^2b + 20\,a^2b.$

18. $25\,ax - 17\,ax - 13\,ax + 19\,ax + 6\,ax - 20\,ax.$

19. $3(ab)^2 + 9(ab)^2 - (ab)^2 + 7(ab)^2 - 9(ab)^2.$

20. $8(a-b) + 4(a-b) - 6(a-b) - 2(a-b) + 3(a-b).$

21. $7\,y^2z - 4\,y^2z + y^2z - 6\,y^2z + 2\,y^2z.$

22. $5(x+y) - 2(x+y) - 3(x+y) + 8(x+y) - 2(x+y).$

23. $4(a+b)^2 + 10(a+b)^2 - 7(a+b)^2 - 2(a+b)^2 + 5(a+b)^2.$

24. $3\,cd - 2\,cd + 5\,cd + 7\,cd - 3\,cd - 4\,cd.$

25. $9(xy)^3 - 3(xy)^3 + 4(xy)^3 - 5(xy)^3 + 2(xy)^3 - 6(xy)^b.$

32. To add when some terms are dissimilar.

1. Add $x + 3\,y - z$, $x - 2\,y$, $x + 4\,y + 3\,z$.

PROCESS.

$$\begin{array}{r} x + 3\,y - z \\ x - 2\,y \phantom{{}+3z} \\ x + 4\,y + 3\,z \\ \hline 3\,x + 5\,y + 2\,z \end{array}$$

EXPLANATION. For convenience in adding, similar terms are written in the same column, and the simplest form of the sum is obtained by beginning at either the right or left hand column and adding each column separately. The dissimilar terms in the result are connected with their proper signs.

RULE. *Write similar terms in the same column. Add each column separately by finding the difference of the sums of the positive and the negative terms. Connect the results with their proper signs.*

Find the sum of each of the following:

2.
$$\begin{array}{r} 2\,a - 4\,b \\ 6\,a - 2\,b \\ 2\,a + 3\,b \\ -5\,a - 4\,b \\ \hline \end{array}$$

3.
$$\begin{array}{r} 10\,x + 3\,y + z \\ -5\,x - y \phantom{{}+4z} \\ 2\,x - 2\,y + z \\ x + 7\,y - 4\,z \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3\,xy + 2\,y^2 - z^2 \\ -2\,xy + 6\,y^2 - 5\,z^2 \\ 7\,xy - 4\,y^2 + 4\,z^2 \\ 2\,y^2 + 5\,z^2 \\ \hline \end{array}$$

5. Find the sum of $2c+5d$, $7c-d$, $d-4c$, $2d-c$.

6. Find the sum of $6m-4n$, $2m+3n$, $5n-7m$, $2n-3m$.

Express in the simplest form:

7. $2a+2b+3c+4b-4a+6a-2c$.

8. $x^2z+5xz^2-7xy+6xz^2-2x^2z+4xy+4x^2z-xz^2$.

9. $3w+4x-7y+2v-2w-x+3y+4v-3x+4w-6v$.

10. $a^2b^2+c^2+cd-2c^2-3cd+5a^2b^2+cd-3c^2-2a^2b^2-c^2$.

11. Add $ab+a^2c-5$, $3ab-3a^2c+7$, $2a^2c-2ab-3$.

12. Add $5a+3b-2c+d$, $2b+c-3d$, $7a-5b+c$.

13. Add $6m+8n+x-y$, $2m-2n+3x+4y$, $-m-5x+2y$.

14. Add $3x+7y-4z+6w$, $7z-4x-2y-5w$, $x+y+z+w$.

15. Add $4x^2y-3xy^2-2x^2y^2$, $4xy^2-3x^2y-2x^2y^2$, $4x^2y^2-2x^2y-3xy^2$.

16. Add $3x^m+2y^n$, $-4x^m+5y^n$, $5x^m-4y^n$, $7x^m-2y^n$.

17. Add $8a^2b^2x^c-3ab+ed$, $2a^2b^2x^c+ab-4ed$, $2ab-a^2b^2x^c$.

18. Add $7m^2-6mn+5n^2$, $4mn-3m^2-n^2$, $5m^2-4n^2+5mn$.

19. Add $14ax^3-8ay^3+6az^3$, $20ay^3-24ax^3-12az^3$, $32ax^3-40ay^3+15az^3$.

20. Add $10a^2b-12a^3bc-15b^2c^4+10$, $-4a^2b+8a^3bc-10b^2c^4-4$, $2a^2b+12a^3bc+5b^2c^4+2$.

21. Add $4x^3-6ax^2+5a^2x-5a^3$, $3x^3+4ax^2+2a^2x+6a^3$, $-17x^3+19ax^2-15a^2x+8a^3$.

22. Add $6(c+d)-3(c+d)+a(c+d)-2b(c+d)-(c+d)$.

23. Add $7a-3b+c+m$, $3b-7a-c+m$.

24. Add $8ax+2(x+a)+3b$, $9ax+6(x+a)-9b$, $11x+6b-7ax-8(x+a)$.

25. Add $6(x+y)+3z-8$, $2(x+y)-2z+4$, $8z-3(x+y)$.

33. A letter may sometimes represent some definite number.

Thus, a may represent 5; then $2a$ will represent 10; $3a$, 15, etc.

A letter may also represent any number, whatever its value.

Thus, 5 times n may represent 5 times any number; 8 times n or $8n$ may stand for 8 times any number.

34. Letters used to represent quantities having a definite value, or letters which represent any number or quantity are called **Known Numbers or Quantities.**

The *first letters* of the alphabet, as a, b, c, etc., are used to represent *known numbers or quantities.*

Thus, a, b, c, d, etc., are usually considered known quantities; that is, they either stand for known numbers or for any number.

PROBLEMS.

35. 1. A man bought a barrels of flour, b barrels of sugar, and 8 barrels of molasses. How many barrels in all did he buy?

2. Edith bought a ribbon for m cents, a pencil for d cents, and a book for 6 cents. How many cents did she pay for all?

3. A farmer sold some sheep for c dollars, a cow for n dollars, and a horse for as much as he received for the sheep and cow. How much did he receive for all?

4. George walked a miles, he then rode 3 miles on his bicycle, and b miles on the cars. How far did he travel?

5. A man began business with $2c$ dollars. The first year he gained $\frac{1}{2}$ as much as he had; the second year $\frac{1}{3}$ as much as he had at the end of the first year; and the third year \$400. How much did he gain in the three years?

6. The letter b represents an odd number. What will represent the next even number? What the next odd number?

7. Laura is m years old; Lizzie is twice as old as Laura; and Mabel's age is equal to $\frac{1}{2}$ the ages of the other two. What is the sum of their ages?

8. A merchant took in c dollars one week, d dollars the next, and \$75 the next. How many dollars did he receive in the three weeks?

9. What is the sum of $x + x + x +$ etc. taken seven times? Of $x + x + x +$ etc. taken a times?

10. A grocer sold b pounds of sugar, c pounds of coffee, d pounds of tea, and 2 pounds of chocolate. How many pounds of groceries did he sell?

11. A man paid m dollars for a coat, n dollars for a waistcoat, p dollars for trousers, g dollars for a pair of boots, and r dollars for a hat. How much did his outfit cost him?

12. A man paid a dollars for a farm; he then expended upon improvements d dollars, and sold it for b dollars more than the entire cost. How much did he receive for it?

13. My fare to San Francisco was a dollars, my sleeping-car charges c dollars, my meals cost me b dollars, and my other expenses \$25. How much did I expend before I reached San Francisco?

SUBTRACTION.

36. 1. What is the difference between 8 days and 5 days?

2. What is the difference between 11 cents and 4 cents?

3. What is the difference between $12d$ and $5d$?

4. What is the difference between $15c$ and $8c$?

5. What is the difference between $12f$ and $8f$?

6. Subtract:

$8x$ from $13x$; $4y$ from $15y$; $6c$ from $13c$; $8d$ from $20d$; $10x^2y$ from $14x^2y$; $8x^3y^2$ from $21x^3y^2$; $4xy^2$ from $14xy^2$; $10a^2b^2$ from $18a^2b^2$; $8a^2x^2y^2$ from $15a^2x^2y^2$; $4abcd$ from $13abcd$; $6abcx$ from $18abcx$; $10ax^2y^2z^2$ from $20ax^2y^2z^2$.

7. What is the remainder when $4a^2x^2y^2$ is subtracted from $10a^2x^2y^2$? What is the sum of $10a^2x^2y^2$ and $-4a^2x^2y^2$?

8. What is the remainder when $3abc^2$ is subtracted from $12abc^2$? What is the sum of $12abc^2$ and $-3abc^2$?

9. Instead of subtracting a positive quantity, what may be done to secure the same result?

10. What is the result when 8 is subtracted from 15? What, when $8-5$ is subtracted from 15?

11. Why is the result 5 more in the latter case than in the former?

12. What is the result when $10a$ is subtracted from $18a$? What, when $10a - 8a$ is subtracted from $18a$?

13. Why is the result greater by $8a$ in the latter case than in the former?

14. Instead of subtracting a negative quantity, what may be done to secure the same result?

15. One vessel was 40 miles east and another 20 miles west from a given meridian. Indicate their relations by proper signs. How far apart were they?

16. Mr. A's property is worth \$$25a$ and Mr. B is \$$8a$ in debt. Indicate their financial conditions by proper signs. What was the difference in their financial condition?

17. A thermometer indicated a temperature of 35° *above* zero on Jan. 5, and of 15° *below* zero on Jan 6. Indicate the temperature by proper signs. What was the difference in temperature?

37. PRINCIPLES. 1. *The difference between similar quantities, only, can be expressed in one term.*

2. *Subtracting a positive quantity is the same as adding a numerically equal negative quantity.*

3. *Subtracting a negative quantity is the same as adding a numerically equal positive quantity.*

38. To subtract when the terms are positive.

1. From $10a$ subtract $4a$.

PROCESS.

$$\begin{array}{r} 10a \\ 4a \\ \hline 6a \end{array}$$

EXPLANATION. When four times any number is taken from ten times that number, the remainder is six times the number; therefore, when $4a$ is subtracted from $10a$, the remainder is $6a$. Or, since subtracting a positive number or quantity is the same as adding an equal negative quantity (Prin. 2), $4a$ may be subtracted from $10a$ by changing the sign of $4a$ and adding the quantities. Therefore, to subtract $4a$ from $10a$, we find the sum of $10a$ and $-4a$, which is $6a$.

2. From $13m$ take $15m$.

PROCESS.

$$\begin{array}{r} 13m \\ 15m \\ \hline -\ 2m \end{array}$$

EXPLANATION. After subtracting from $13m$ as much as we can of $15m$, there will be $2m$ yet to be subtracted, or the result will be $-2m$. Or, since subtracting a positive number or quantity is the same as adding an equal negative quantity (Prin. 2), $15m$ may be subtracted from $13m$ by finding the sum of $13m$ and $-15m$, which is $-2m$. Therefore, when $15m$ is taken from $13m$, the result is $-2m$.

	3.	4.	5.	6.	7.	8.
From	$19x$	$7ab$	$18m^2$	$16xy$	$6x^2y^2z$	$9mn$
Take	$4x$	$3ab$	$13m^2$	$20xy$	$8x^2y^2z$	$14mn$

Subtract the following:

9. $8x+3y$ from $12x+7y$.

10. $5ab+3c$ from $10ab+2c$.

11. $4b+9d$ from $8b+5d$.

12. $5x+y$ from $8x+3y$.

13. $4x+8z$ from $5x+9z$.

14. $3a+2b+5c$ from $7a+5b+6c$.

15. $7a+2b$ from $9a+b$.

16. $5x^2+8y^2+6xy$ from $7x^2+3y^2+2xy$.

17. $4xy+3z$ from $10xy+5z$.

18. $5(a^2+b^2)+7c^2+2d$ from $6(a^2+b^2)+4c^2+4d$.

19. $3a(p+q)+2$ from $6a(p+q)+7$.

20. $5x+7y+4z+5$ from $5x+4y+8z+6$.

21. $3x^2+2y+z+4v$ from $7x^2+3y+6z+v$.

22. $2x+5(y+z)+8$ from $7x+2(y+z)+10$.

23. $3a^2+5b^2+4c^2+d^2$ from $a^2+6b^2+c^2+5d^2$.

39. To subtract when some terms are negative.

1. From $7x - 2y$ subtract $4x - 3y$.

PROCESS.

$$\begin{array}{r} 7x - 2y \\ 4x - 3y \\ - \quad + \\ \hline 3x + \ y \end{array}$$

EXPLANATION. Since the subtrahend is composed of two terms, each term must be subtracted separately.

Subtracting $4x$ from $7x - 2y$, leaves $3x - 2y$, or the result may be obtained by adding $-4x$ to $7x - 2y$. But since the subtrahend was $3y$ less than $4x$, to obtain the true remainder, $3y$ must be added to $3x - 2y$, which gives $3x + y$.

Therefore the subtraction may be performed by changing the sign of each term of the subtrahend and adding the quantities.

RULE. *Write similar terms in the same column. Change the sign of each term of the subtrahend from + to −, or from − to +, or conceive it to be changed, and proceed as in addition.*

	2.	3.	4.	5.	6.
From	$4a^3x$	$9x^2yz$	$6y - 4z$	$7x^2 + 5y^2$	$3a - 3b + c$
Take	$3a^3x$	$-7x^2yz$	$5y + 2z$	$4x^2 - 8y^2$	$3a + 3b - c$

	7.	8.	9.
From	$4ab - 3c + d$	$5x^2 - 3xy + x$	$10ax - 13y + z^3$
Take	$-2ab + 5c - d$	$6x^2 + 5xy - x$	$5ax - 7y - z^3$

10. From $12x^5 - 20x^3y^3 + 6xy$ subtract $9x^3y^3 - 2xy + 4x^5$.

11. From $4ab + 3b^2 - 6cd$ subtract $6ab - 2b^2 + 2cd$.

12. From $9x^2y - 6xy^2 - 2xy + 5$ subtract $3x^2y + xy + 8$.

13. From $7x^m + 2x^my^n - 5y^n$ subtract $4x^m - 2x^my^n - 9y^n$.

14. From $8(m + n^2) - 12(m^2 + n)$ subtract $12(m + n^2) - 8(m^2 + n)$.

15. From $4x^3 + 7y^3 - 3z^3 + r^3$ subtract $-2x^3 + y^3 - 3r^3$.

16. From $5(p + q) - 6(r + s) + 15$ subtract $8(p + q) + 2(r + s) + 25$.

17. From $18m^2nx^2 + 12a^2bc^2 + abd$ subtract $-2m^2nx^2 + 5abd$.

18. From $a^2 - 2ab + c^2 - 3b^2$ subtract $2a^2 - 2ab + 3b^2$.

19. From $2x^2 + 2y^2 - 4xy$ subtract $2x^2 + 2y^2 + 4xy$.

20. From $4x^4 + 3x^2y^2 - 8y^4$ subtract the sum of $x^4 - x^2y^2 + 3y^4$ and $2x^4 + 2x^2y^2 - 9y^4$.

21. From $5x^3y^3 + 10x^4y - 6yz^3$ subtract $10x^4y - 4x^3y^3 + 5yz^3$.

22. From $3x^m - 4x^ny^m + 4y^m$ subtract $4x^m + 2x^ny^m - y^m$.

23. From $4bx^3 + 3ay^2 + 4 - cy$ subtract $cy - 5 - bx^3$.

24. From $10m^2 - 4mn - 3n^2 - 18$ subtract the sum of $m^2 - 3mn + 4$ and $5m^2 - 2mn + 6n^2$.

25. From $a^2x - x^2 + x^3y - 8$ subtract $3a^2x - 5 - x^2 + 2x^3y$.

26. From $15x^2y^2 - 15$ subtract $4x^2y^2 + z^2 - 4y^2 - 10$.

27. From $2x + y + z + u$ subtract $x + 2y + 2z + 2u$.

28. From $16a^2b^2 - 12bc + 14b^2$ subtract the sum of $a^2b^2 - 3bc + 4$ and $10a^2b^2 + 8bc + 5b^2$.

29. From $ax + by$ subtract $cx - dy$.

PROCESS.

$$\begin{array}{l} ax + by \\ cx - dy \\ \hline (a-c)x + (b+d)y \end{array}$$

EXPLANATION. Since a and c may be regarded as the coefficients of x, and b and $-d$ the coefficients of y, the difference between the quantities may be found by writing the difference between the coefficients of x and y respectively for the coefficients of the remainder. Since c cannot be subtracted from a, the subtraction is indicated by $(a - c)$, and since $-d$ cannot be subtracted from b, the subtraction is indicated by $(b + d)$, consequently the remainder may be written $(a - c)x + (b + d)y$.

30. From $ax + 2y$ subtract $2x - by$.

31. From $2cx - ay + 3z$ subtract $cy + 5x - az$.

32. From $cx - 12aby + 4a^2b^2$ subtract $6x - 10aby + 3b^2$.

33. From $2a(x - y) + 4abx$ subtract $3c(x - y) + 2ax$.

34. From $ax + by + cz$ subtract $bx + cy + dz$.

35. From $3x + 7y + 8z$ subtract $bcx - ay + az$.

36. From $5ay + 2cz + 6x$ subtract $cy - az - dx$.

37. From $(a - b)x + (c + d)y$ subtract $ax + dy$.

38. From $7x^2 + 5ay^2 + 6z^2$ subtract $2ax^2 + 3y^2 - abz^2$.

39. From $(m+n)x^2 + (m-n)y^2 + z^2$ subtract $mnx^2 - 4y^2 + az^2$.

40. From $a(x + y) + b(x - y) + dx$ subtract $b(x + y) - a(x - y) - cx$.

41. Express the difference between m and n.

42. Write the number one less than x; the number one greater.

43. A man sold a horse for \$125 and gained a dollars. What did the horse cost?

44. A girl earns b cents a day and spends 25 cents a week. How much has she left at the end of the week?

45. The sum of two numbers is 30, and x represents one of them. What represents the other?

46. The difference between two numbers is 5. How may the numbers be represented?

47. A merchant bought a hat for b dollars and a coat for c dollars, and sold the two for d dollars. What represents his gain?

48. A man whose income is a dollars spends m dollars for rent, n dollars for living expenses, and 100 dollars for other expenses. What represents the amount he saves?

49. A lady paid a dollars for a dress, c dollars for a hat, and \$25 for a cloak. How much had she left from a \$50 bill?

50. A man paid 40 dollars for a cords of wood, and sold it at 3 dollars a cord. How much did he gain?

51. A farmer sold some grain for b dollars, some fruit for d dollars, and some hay for e dollars. He received in part payment a horse worth f dollars. How much remained still to be paid?

40. The parenthesis, ( ), the vinculum, ——, the bracket, [ ], and the brace, { }, are called **Signs of Aggregation.** They show that the quantities included by them are to be treated as a single quantity.

Thus, $(a + b) \times c$, $\overline{a + b} \times c$, and $\{a + b\} \times c$ show that the sum of a and b is to be multiplied by c.

41. The subtrahend is sometimes expressed with a sign of aggregation and written after the minuend with the sign $-$ between them.

Thus, when $b + c - d$ is subtracted from $a + b$, the result is sometimes indicated as follows: $a + b - (b + c - d)$.

1. What change must be made in the signs of the terms of the subtrahend when it is subtracted from the minuend?

2. When a quantity in parenthesis is preceded by the sign $-$, what change must be made in the signs of the terms when the subtraction is performed, or when the parenthesis or other similar sign is removed?

The term parenthesis is commonly used to include all signs of aggregation.

42. Principles. 1. *A parenthesis, preceded by the minus sign, may be removed from an expression if the signs of all the terms in parenthesis are changed.*

2. *A parenthesis, preceded by the minus sign, may be used to inclose an expression if the signs of all the terms to be inclosed in parenthesis are changed.*

When quantities are inclosed in a parenthesis preceded by the plus sign, the parenthesis may be removed without any change of signs, and, consequently, any number of terms may be inclosed in a parenthesis with the *plus* sign without any change of signs.

The student should remember that in expressions like $-(x^2-y+z)$ the sign of x^2 is plus, and the expression is the same as if written $-(+x^2-y+z)$.

Simplify the following:

1. $20-(3+4-6+5)$.

SOLUTION. $20-3-4+6-5=14$.

2. $25-(6+9-3-7+13)$.
3. $5-(3+2-6)-(-2-4+1)$.
4. $(7-5+3)-(8-9+2)$.
5. $15-(3+4-6)-(8-6+7)$.
6. $(4+10-8)-(3+7-4)$.
7. $19-(12-5+8)-(11-5-3)$.
8. $30-(-6+8-1)-(18-10+4)$.
9. $(14-5+2)-(18-20+5)$.
10. $15-(3+8-7)+16-(10+4+5-8)$.
11. $(5+7-8)-6-(-5+3-6+2)$.
12. $16-[8-(5+6-4)+12]$.

SOLUTION.
$$\begin{aligned} & 16-[8-(5+6-4)+12] \\ = & 16-[8-5-6+4+12] \\ = & 16-8+5+6-4-12 \\ = & 3. \end{aligned}$$

13. $25-[13-4+(3-10-2)]$.

14. $10-\{8-(15+7+3)+6\}$.

15. $17-(3+8)-[12-(3+8)-5]$.

16. $-\{-16+13-(6-1+4)+5-10\}$.

17. $(3+7-4)-[14-(13+7)+5]$.

Simplify the following:

18. $a-(b-c+d-e)$.

SOLUTION. $a-(b-c+d-e)=a-b+c-d+e$.

19. $2x-(x-5x+3x-8x)$.

20. $7m-(3n+2m-6m+n)$.

21. $4x^2+7ax-(5ax+3ax-2x^2+10ax)$.

22. $a^2+b^2-(-2ab-2a^2-2b^2)$.

23. $(6xy+2z)-(4z+3xy-2z+5)$.

24. $a-b-(a+b-c-3)$.

25. $a+b-(2a-3b)-(5a+7b)-(-13a+2b)$.

26. $(a+b+c)+(-a+b-c)-(a-b+c)$.

27. $x-[-x+2x-(x+2x)-2x]$.

28. $3x-[x-3z-(2y-z)]$.

29. $a^2-a-(4a-y-3a^2-1)$.

30. $m+n-(m+n)-\{m-n-(m+n)-n\}$.

31. $(x^2+2xy+y^2)-(2xy-x^2-y^2)$.

32. $9x-[8x-\overline{6x-3x}]$.

33. $(x+10)-\{x-\overline{3x+25}-10\}$.

34. $8a-(6a-5)-(5a+11-4a)$.

35. $a-[2b-(3c+2b)-a]$.

TRANSPOSITION IN EQUATIONS.

43. 1. If a certain number, diminished by 3, equals 15, what is the number? If $x-3=15$, what is the value of x?

2. If a certain number, increased by 3, equals 15, what is the number? If $x+3=15$, what is the value of x?

3. In the equation $x-3=15$, what is done with the 3 in obtaining the value of x? In the equation $x=15+3$, how does the sign of 3 compare with its sign in the original equation?

4. In the equation $x+3=15$, what is done with the 3 in obtaining the value of x? In $x=15-3$, how does the sign of 3 compare with its sign in the original equation?

5. In changing the 3's from one side, or *member*, of the equation to the other, what change was made in the sign?

6. When a number or quantity is changed from one member of an equation to the other, what change must be made in its sign?

7. If any number, as 5, is added to one member of the equation $2+3=5$, what must be done to the other member to preserve the equality?

8. If any number, as 3, is subtracted from one member of the equation $2+3=5$, what must be done to the other member to preserve the equality?

9. If one member of the equation $2+3=5$ is multiplied by any number, as 4, what must be done to the other member to preserve the equality?

10. If one member of the equation $2+3=5$ is divided by any number, as 5, what must be done to the other member to preserve the equality?

11. If one member of the equation $5 + 3 = 8$ is raised to any power, as the second power, what must be done to the other member to preserve the equality?

12. What, then, may be done to the members of an equation without destroying the equality?

44. The parts on each side of the sign of equality are called the **Members of an Equation.**

45. The part of an equation on the left of the sign of equality is called the **First Member.**

46. The part of an equation on the right of the sign of equality is called the **Second Member.**

47. The process of changing a term from one member of an equation to another is called **Transposition.**

48. A truth that does not need demonstration is called an **Axiom.**

Axioms. 1. *Things that are equal to the same thing are equal to each other.*

2. *If equals are added to equals, the sums are equal.*

3. *If equals are subtracted from equals, the remainders are equal.*

4. *If equals are multiplied by equals, the products are equal.*

5. *If equals are divided by equals, the quotients are equal.*

6. *Equal powers of equal quantities are equal.*

49. Principle. *A term may be transposed from one member of an equation to the other if its sign is changed from + to −, or from − to +.*

EQUATIONS AND PROBLEMS.

50. 1. Given $2x - 3 = x + 6$, to find the value of x.

PROCESS.

$$\begin{array}{rl} 2x - 3 &= x + 6 \\ +3 &= \quad +3 \\ \hline 2x \quad &= x + 9 \\ x \quad &= x \\ \hline x \quad &= 9 \end{array}$$

OR,

$$2x - 3 = x + 6$$
$$2x - x = 6 + 3$$
$$x = 9$$

VERIFICATION.

$$18 - 3 = 9 + 6$$
$$15 = 15$$

EXPLANATION. Since the known and the unknown quantities are found in both members of the equation, to find the value of x, the known quantities must be collected in one member and the unknown in the other.

Since -3 is found in the first member, it may be caused to disappear by adding 3 to both members (Ax. 2), which gives the equation, $2x = x + 9$.

Since x is found in the second member, it may be caused to disappear by subtracting x from both members (Ax. 3), which gives as a resulting equation, $x = 9$.

Or, since a term may be changed from one member of an equation to the other by changing its sign (Prin.), -3 may be transposed to the second member by changing it to $+3$, and x may be transposed to the first member by changing it to $-x$. Then, the resulting equation will be $2x - x = 6 + 3$.

By uniting the terms, $x = 9$.

The result may be *verified* by substituting the *value* of x for x in the original equation.

If both members are then *identical*, the value of the unknown quantity is correct. Thus, if 9 is substituted for x in the original equation, the equation becomes $18 - 3 = 9 + 6$, or $15 = 15$.

Therefore, 9 is the correct value of x.

RULE. *Transpose the terms so that the unknown terms stand in the first member of the equation, and the known terms in the second.*

Unite similar terms, and divide each member of the equation by the coefficient of the unknown quantity.

VERIFICATION. *Substitute the value of the unknown quantity for the quantity in the original equation. If both members are then identical in value, the value of the unknown quantity found is correct.*

Transpose, and find the value of x:

2. $x+4=10$.
3. $x-3=4$.
4. $2x+1=5$.
5. $6x-6=12$.
6. $4x+3=15$.
7. $8x-2=14$.
8. $3x+5=26$.
9. $9x-5=31$.
10. $5x+2=10+7$.
11. $7x-1=30+4$.
12. $2x-10=3+5$.
13. $4x-2x=3+7$.
14. $6x-3=2x+9$.
15. $5x-15+2=2-3x+1$.
16. $6x-2x+4=16-8$.
17. $7x-3=2x-4+11$.
18. $9x-10=2x+4$.
19. $6x+25=88-x$.
20. $3x-4=12-4$.
21. $5x-5=67-3x$.
22. $10x-20=24-12x$.
23. $3x-14=10-x$.
24. $2x-16=20-4x$.
25. $15x-39=29-2x$.
26. $3x-18=31-4x$.
27. $4x-14=49-3x$.
28. $5x-20=25-4x$.
29. $2x-36=60-6x$.
30. $3x-20+x=44-4x$.
31. $5+8x-7=3x+3$.
32. $9x-15=6+7x+3$.
33. $9x+5-15=5x+2x$.
34. $8x-10=10+2x+4$.
35. $4x-6x-6=6+3-3x$.
36. $x-5=18-4x-3$.
37. $3x+6x=18-x+2$.
38. $3x-6=x+14-4$.
39. $9x+13=26+2x+1$.
40. $4x+4-3x=16-2x$.
41. $27x-14=190-41x$.
42. $6x-12=4x+18$.
43. $5x-15=3x+9$.
44. $18x+9=15x+30$.
45. $7x-3+2x=5x+20+1$.
46. $6x-21=x+14-2x$.
47. $9x+3-24=5x-5$.
48. $10x-4+3=6x+19$.
49. $3x-15-10=20-2x$.
50. $5x-5-20+6x=41$.
51. $2x-25=35-x-3x$.
52. $3x-19=20-10x+13$.
53. $5x-16=25-x+40-3x$.
54. $6x-5-30=10-4x-5x$.
55. $7x-30=10+16-7x$.
56. $5x-50=25-5x+25$.
57. $10x-22=17-2x-x$.

PROBLEMS.

51. 1. Twice a certain number increased by 15 is equal to the number increased by 19. What is the number?

2. What number is that whose double exceeds the number by 12?

3. Ten times a certain number diminished by 13 is equal to the number plus 5. What is the number?

4. What number diminished by 8 equals 6?

5. Six times a certain number plus 7 equals five times the number plus 12. What is the number?

6. Three boys together had 90 cents. The first had 10 cents more than the second, and the second had 1 cent more than the third. How much had each?

7. A father gave a certain sum to his youngest son, and 4 cents more to the next older, and 10 cents to the oldest. If he gave to all 20 cents, how much did he give to each?

8. The greater of two numbers exceeds the less by 14, and the sum of the numbers is 34. What are the numbers?

9. A and B started in business, A furnishing $4000 more than B. Three times B's capital was then equal to A's. How much did each furnish?

10. A and B had the same sum of money. A gave B $4, and then B had double the amount A had left. How much had each at first?

11. A tourist rode 32 miles upon a bicycle. A certain number of miles was down hill, twice as far plus 8 miles was level, and the distance up hill was $\frac{1}{2}$ as far as the distance on a level. How many miles did he travel upon each kind of road?

12. A man has two horses, of unequal value, together worth $200. If he should put a saddle worth $30 on the poorer horse, the horse and saddle would together be equal in value to the better horse? What is the value of each?

13. Six hundred gallons of water are discharged into a cistern by 3 pipes. The second discharges 100 gallons more than the first, and the third discharges three times as much as the first. How many gallons are discharged by each pipe?

14. A drover being asked the number of his cattle said that if he had three times as many as he then had and 25 more, he would have 1000. How many cattle had he?

15. A man bought a watch and chain for $60. The watch cost 12 times as much as the chain lacking $5. What was the cost of each?

16. A tenement house contained 90 persons, men, women, and children. If there were 4 more men than women, and 10 more children than men and women together, how many were there of each?

17. A steamer and its cargo are together worth $120,000. If the steamer lacks only $8400 of being worth twice as much as the cargo, what is the value of each?

18. A clerk's expenses are $400 per year, and his brother's are $600 per year. If the brother has three times as large an annual salary and he has left at the end of the year a sum equal to twice his brother's salary, what is the salary of each?

19. If a house and lot cost four times as much as the lot, and the house cost $2500 more than twice as much as the lot, what was the cost of each?

MULTIPLICATION.

52. 1. What is the sum of $5m + 5m + 5m$? Or, how much is 3 times $5m$? 8 times $5m$?

2. What is the sum of $8xy + 8xy + 8xy + 8xy$? Or, how much is 4 times $8xy$? 10 times $8xy$?

3. How much is 6 times $7bc$? Which quantity is the multiplier? Which is the multiplicand? What sign has the multiplier? What sign has the multiplicand? What sign has the product?

4. When a *positive* quantity is multiplied by a *positive* quantity, what is the sign of the product?

5. If a vessel sails south 8 miles per hour, indicated by -8 mi., how far will she sail in 5 hours? What will be the sign of the product?

6. How much is 4 times $-5xy$? 6 times $-6ab$? 7 times $-8cd$? What is the sign of the multiplier in each case? What is the sign of the multiplicand? What is the sign of the product? $-4ab \times 8 = ?$

7. When a *negative* quantity is multiplied by a *positive* quantity, what is the sign of the product?

8. How does the product of 6 times 7 compare with the product of 7 times 6? What effect upon the product has it to change the order of the factors, when the numbers or quantities are abstract?

9. How, then, will the product of $-3a$ multiplied by $+4$ compare with the product of $+4$ multiplied by $-3a$? What is the product? $-5xy \times 6 = ?$ $6 \times -5xy = ?$ $-7ab \times 4 = ?$ $4 \times -7ab = ?$

10. When a *positive* quantity is multiplied by a *negative* quantity, what is the sign of the product?

11. How much is 6 times $-3a$? 2 times $-3a$? $(6-2)$ times $-3a$, or 6 times $-3a$, -2 times $-3a$?

12. Since 2 times $-3a$, or $-6a$, must be subtracted from $-18a$ to obtain the correct product, what will be the sign of $-6a$ after it is subtracted?

13. Since -2 times $-3a$ gives a product of $+6a$, what may be inferred regarding the sign of the product when a *negative* quantity is multiplied by a *negative* quantity?

14. What is an exponent? What does it show? What does a^5 mean? When a^5 is multiplied by a^2, how many times is a used as a factor to obtain the product? How many times, when a^4 is multiplied by a^5?

15. How, then, may the number of times a quantity is used as a factor in multiplication be determined from the exponents of the quantities in the expressions multiplied? How may the exponent of a quantity in the product be determined?

16. $3a^2 \times 6 = ?$ $10a^5 \times 5 = ?$ $20a^3 \times 3 = ?$ $25x^2y \times 2 = ?$ How is the coefficient of the product determined from the coefficients of the factors, or from the multiplier and the multiplicand?

53. Multiplication is indicated in four ways:

1. By the sign $\times$, read *multiplied by* or *times.*

Thus, $a \times b$ shows that a is to be multiplied by b.

2. By the dot (·), read *multiplied by* or *times.*

Thus, $a \cdot b$ shows that a is to be multiplied by b.

3. By writing letters, or a number and a letter side by side.

Thus, ab shows that a is to be multiplied by b; and $5a$ shows that a is to be multiplied by 5.

4. By a small figure or letter, called an *Exponent,* written a little above and at the right of a quantity, showing the number of times the quantity is to be used as a factor.

Thus, a^5 shows that a is to be used as a factor 5 times, or that it is equal to $a \times a \times a \times a \times a$ or $aaaaa$.

54. PRINCIPLES. 1. *The sign of any term of the product is + when its factors have* LIKE *signs, and − when they have* UNLIKE *signs.*

2. *The coefficient of a quantity in the product is equal to the product of the coefficients of its factors.*

3. *The exponent of a quantity in the product is equal to the sum of its exponents in the factors.*

55. To multiply when the multiplier is a monomial.

1. What is the product of $5x^2yz$ multiplied by $3abx$?

PROCESS.

$$\begin{array}{r} 5x^2yz \\ 3abx \\ \hline 15abx^3yz \end{array}$$

EXPLANATION. The coefficient of the product is obtained by multiplying 5 by 3 (Prin. 2). The literal quantities are multiplied by adding their exponents (Prin. 3). Hence, the product is $15abx^3yz$.

2. What is the product of $3b - c$ multiplied by $5c^2$?

PROCESS.

$$\begin{array}{r} 3b - c \\ 5c^2 \\ \hline 15bc^2 - 5c^3 \end{array}$$

EXPLANATION. The product of $3b$ multiplied by $5c^2$ is $15bc^2$. But, since the *entire* multiplicand is $3b - c$, the product of c multiplied by $5c^2$ must be subtracted from $15bc^2$. The product of c multiplied by $5c^2$ is $5c^3$, which subtracted from $15bc^2$ gives the entire product $15bc^2 - 5c^3$.

RULE. *Multiply each term of the multiplicand by the multiplier, as follows:*

To the product of the numerical coefficients, annex each literal factor with an exponent equal to the sum of the exponents of that letter in both factors.

Write the sign + before each term of the product when its factors have like signs, and − when they have unlike signs.

	3.	4.	5.	6.	7.	8.	9.	10.
Multiply	10	$10a$	$-6a$	$22x$	18	$-14b$	$17c$	$24x$
By	4	-4	3	-5	$6x$	3	$3c$	$-9x$

	11.	12.	13.	14.	15.
Multiply	$5x^2yz$	$-15xy^2$	$16c^2dz$	$-42m^2n^2$	$-25a^2b$
By	$2x^2yz$	$-3xy^2$	$-4dz$	$-3mn$	$5ab^2$

	16.	17.	18.	19.
Multiply	$(a+b)$	$4(x-y)$	$3x+2y-5z$	$2x^2y-2y^2z$
By	4	-5	$2a$	$3x$

Multiply:

20. $3a^2x - 5x^2y + 2y^2$ by $-4xy$.

21. $a^2 + 2ab + b^2$ by ab.

22. $6m + 7mn + 5n^2$ by $-3mn$.

23. $x^4 - 2x^3 + 5x^2 + x - 3$ by $9x^2$.

24. $9a^2 - 13ab + 4b^2 - 6$ by $12a^2b^2$.

25. $x^2 - 3xy - 3xz + y^2z^2$ by $2ax$.

26. $4ab + 3a^2b - 5ab^2 - 2a^2$ by $3b^2$.

27. $3m^2 - 10mn - 8n^2$ by $4mn$.

28. $x^4 + x^3 + x^2 + x + 1$ by $-5x$.

29. $6a^2 - 18ab + 15c^2 - 20abc + 14$ by $3a^2b^2$.

56. To multiply when the multiplier is a polynomial.

1. Multiply $a+b$ by $a+b$.

SOLUTION.

$$\begin{array}{rl} & a+b \\ & \underline{a+b} \\ a \text{ times } a+b = & a^2+ab \\ b \text{ times } a+b = & \underline{\quad\;\; ab+b^2} \\ (a+b) \text{ times } (a+b) = & a^2+2ab+b^2 \end{array}$$

RULE. *Multiply each term of the multiplicand by each term of the multiplier, and add the partial products.*

2.

$$\begin{array}{l} 2ab-3c \\ \underline{4ab+c} \\ 8a^2b^2-12abc \\ \underline{\qquad\qquad 2abc-3c^2} \\ 8a^2b^2-10abc-3c^2 \end{array}$$

3.

$$\begin{array}{l} x^3-3x^2y+3xy^2-y^3 \\ \underline{x-y} \\ x^4-3x^3y+3x^2y^2-xy^3 \\ \underline{\quad -x^3y+3x^2y^2-3xy^3+y^4} \\ x^4-4x^3y+6x^2y^2-4xy^3+y^4 \end{array}$$

Multiply:

4. $x+y$ by $x+y$.
5. $m+n$ by $m-n$.
6. $a^2+2ab+b^2$ by $a+b$.
7. $2a-5b$ by $2a+5b$.
8. $x+4$ by $x-10$.
9. $3y+2z$ by $2y+3z$.
10. $3a+7b$ by $3a-7b$.
11. $2x+1$ by $3x-6$.
12. $2a-3b$ by $3a+5b$.
13. $3m+4n$ by $2m+3n$.
14. $5y-3z$ by $4y-4z$.
15. $2b-5c$ by $3b+8c$.
16. $3x-20$ by $8x+4$.
17. $5m-4n$ by $4m+5y$.
18. $x+2y$ by $x+5y$
19. $1+x+x^2$ by $1-x$.
20. $x+y+1$ by $x-y-1$.
21. $2x+4y$ by $3x-2y$.
22. $a+b-2c$ by $2a-b$.
23. $4x+7$ by $3x-2$.
24. $3am+6c$ by $8ac+c^2$.
25. $3xy-6y$ by $4xy+8y$.
26. $x+y-z$ by $x+y$.
27. $a-b-c$ by $a-c$.
28. $2a+x-y$ by $a-x$.
29. $2x+3y-6$ by $x+5y$.

30. $3xz + 2y^2$ by $8xz - 4y^2$.

31. $m + n + 1$ by $m - n + 1$.

32. $5x + 4$ by $5x - 9$.

33. $3x^2 - 4y^2$ by $3x^2 - 7y^2$.

34. $a^3 + 3a^2b + 3ab^2 + b^3$ by $a + b$.

35. $4x^2 - 12xy + 9y^2$ by $2x - 3y$.

36. $m^4 + m^3n + m^2n^2 + mn^3 + n^4$ by $m - n$.

PROBLEMS.

57. **1.** If from three times a number 4 is subtracted and the remainder is multiplied by 6, the result is 12. What is the number?

2. If from two times a number 4 is subtracted and the remainder is multiplied by 3, the result equals two times the sum of that number and 2. What is the number?

3. A father is four times as old as his son, and 5 years ago he was seven times as old as his son. What is the age of each?

4. Samuel and John together have 40 cents. If John had 5 cents less, and Samuel 5 cents more, Samuel would have three times as much money as John. How many cents has each?

5. A commenced business with three times as much capital as B. During the first year A lost $\frac{1}{3}$ of his money, and B gained $500. The amount of A's and B's money was then equal. How much had each at first?

6. A is 50 years of age; B is 10. When will A be three times as old as B?

7. Six men hired a boat, but 2 of them being unable to pay their share, the other 4 were obliged to pay 1 dollar more each. For how much did they hire the boat?

8. Three times the difference between a certain number and 10 equals two times the sum of the number and 10. What is the number?

9. Express the product of the factors $2, x, y, z, x^2, y, 4z$.

10. What will d quarts of milk cost at f cents per quart?

11. How far will a man travel in a hours if he goes $b + 6$ miles per hour?

12. A farmer has a cows and three times as many sheep less 8. How many animals does he own?

13. A man sold 20 acres of land at a dollars per acre. With a part of the money he bought 3 horses at d dollars each. How much money had he left?

14. If a men can do some work in 12 days, how long will it take one man to do the same work?

15. A starts in business with b dollars; B starts with c dollars. In one year A gains as much more, while B gains $\frac{1}{2}$ as much more. How much has each at the end of the year?

16. What will 10 bushels of potatoes cost at $2m$ cents per bushel?

17. A man earns $2 per day and pays $$a$ per week for his board. How much money will he have at the end of b weeks?

18. An engine pumps 150 gallons of water into a tank each day; $10c$ gallons are drawn off each day. How much water will remain in the tank at the end of 4 days?

19. The daily wages of a mechanic are a dollars. How much will the wages of 10 mechanics for c days be?

SPECIAL CASES IN MULTIPLICATION.

58. The square of the sum of two quantities.

$$\begin{array}{r} a+\phantom{2ab+{}}b \\ a+\phantom{2ab+{}}b \\ \hline a^2+ab \\ ab+b^2 \\ \hline a^2+2ab+b^2 \end{array} \qquad \begin{array}{r} m+\phantom{2mn+{}}n \\ m+\phantom{2mn+{}}n \\ \hline m^2+mn \\ mn+n^2 \\ \hline m^2+2mn+n^2 \end{array}$$

1. How is the first term of the second power, or square, of the quantities obtained from the quantities? How is the second term obtained? The third term?

2. What signs have the terms?

59. Principle. *The square of the sum of two quantities is equal to the square of the first quantity, plus twice the product of the first and second, plus the square of the second.*

Write out the products or powers of the following:

1. $(x+y)(x+y)$.
2. $(b+c)(b+c)$.
3. $(m+2)(m+2)$.
4. $(a+x)(a+x)$.
5. $(x+3)(x+3)$.
6. $(y+1)(y+1)$.
7. $(2x+y)(2x+y)$.
8. $(m+2n)(m+2n)$.
9. $(3a+b)(3a+b)$.
10. $(2x+3y)(2x+3y)$.
11. $(a+4b)(a+4b)$.
12. $(2m+2n)(2m+2n)$.
13. Square $2x+5$.
14. Square $3m+1$.
15. Square $2a+5b$.
16. Square a^2+b^2.
17. Square x^2+3.
18. Square $2m^2+3n^2$.
19. Square $ab+2c$.
20. Square $2xy+z$.
21. Square y^2+4z^2.
22. Square x^2+8.
23. Square $5a+7b$.
24. Square $4a^2+3b^2$.

60. The square of the difference of the two quantities.

$$\begin{array}{rr}
x - & y \\
x - & y \\
\hline
x^2 - & xy \\
- & xy + y^2 \\
\hline
x^2 - & 2xy + y^2
\end{array}
\qquad
\begin{array}{rr}
c - & d \\
c - & d \\
\hline
c^2 - & cd \\
- & cd + d^2 \\
\hline
c^2 - & 2cd + d^2
\end{array}$$

1. How is the first term of the second power obtained from the terms of the quantity squared? How is the second term obtained? The third term?

2. What signs connect the terms of the power?

61. PRINCIPLE. *The square of the difference of two quantities is equal to the square of the first quantity, minus twice the product of the first and second, plus the square of the second.*

Write out the products or powers of the following:

1. $(a-x)(a-x)$.
2. $(b-c)(b-c)$.
3. $(m-n)(m-n)$.
4. $(x-2)(x-2)$.
5. $(y-z)(y-z)$.
6. $(a-3b)(a-3b)$.
7. $(b-2c)(b-2c)$.
8. $(2x-2y)(2x-2y)$.
9. $(b-5)(b-5)$.
10. $(y-1)(y-1)$.
11. $(ab-2)(ab-2)$.
12. $(x-4)(x-4)$.
13. Square $2a-3b$.
14. Square $m-2n$.
15. Square $2b-4d$.
16. Square a^2-2b^2.
17. Square $bc-xy$.
18. Square $2x^2-5y^2$.
19. Square $2a-c$.
20. Square $3m^2-1$.
21. Square $3mn-4$.
22. Square y^2-6.
23. Square $4x^2-5y^2$.
24. Square $ab-2c^2$.

62. The product of the sum and difference of two quantities.

$$\begin{array}{ll} x - y & \\ x + y & \\ \hline x^2 - xy & \\ \quad\;\; xy - y^2 & \\ \hline x^2 \qquad - y^2 & \end{array} \qquad\qquad \begin{array}{l} c + d \\ c - d \\ \hline c^2 + cd \\ \quad - cd - d^2 \\ \hline c^2 \qquad - d^2 \end{array}$$

1. How are the terms of the product of the sum and difference of two quantities obtained from the quantities?

2. What sign connects the terms?

63. PRINCIPLE. *The product of the sum and difference of two quantities is equal to the difference of their squares.*

Write the products of the following:

1. $(a+b)(a-b)$.
2. $(m+n)(m-n)$.
3. $(a+x)(a-x)$.
4. $(2a+b)(2a-b)$.
5. $(2x+y)(2x-y)$.
6. $(a+4)(a-4)$.
7. $(2m+3n)(2m-3n)$.
8. $(y+1)(y-1)$.
9. $(x+5)(x-5)$.
10. $(2+y)(2-y)$.
11. $(ab+3c)(ab-3c)$.
12. $(2m+2n)(2m-2n)$.
13. $(b+2c)(b-2c)$.
14. $(3x+8y)(3x-8y)$.
15. $(x+10)(x-10)$.
16. $(bc+ef)(bc-ef)$.
17. $(3x^2+2y^2)(3x^2-2y^2)$.
18. $(5a+3x)(5a-3x)$.
19. $(a^2+b^2)(a^2-b^2)$.
20. $(mn+4)(mn-4)$.
21. $(x+6)(x-6)$.
22. $(4y+7z)(4y-7z)$.
23. $(3x+4y)(3x-4y)$.
24. $(2ab+5c)(2ab-5c)$.

64. The product of two binomials.

$$\begin{array}{lll}
x+3 & x+3 & x-3 \\
x+5 & x-5 & x-5 \\
\hline
x^2+3x & x^2+3x & x^2-3x \\
\quad\;\; 5x+15 & \quad -5x-15 & \quad -5x+15 \\
\hline
x^2+8x+15 & x^2-2x-15 & x^2-8x+15
\end{array}$$

1. How is the first term of each product obtained from the factors?

2. How is the second term of the product in the first example obtained from the factors? The second term in the second example? The second term in the third example?

3. How is the third term of the product in each example obtained from the factors?

4. How are the signs determined which connect the terms?

65. Principle. *The product of two binomial quantities having a common term is equal to the square of the common term, the algebraic sum of the other two multiplied by the common term, and the algebraic product of the unlike terms.*

Write the products of the following:

1. $(x+3)(x+4)$.
2. $(x-1)(x+5)$.
3. $(x-2)(x-3)$.
4. $(x+6)(x-7)$.
5. $(x+5)(x+10)$.
6. $(x-13)(x+3)$.
7. $(x+20)(x+5)$.
8. $(x-4)(x+8)$.
9. $(x+9)(x+3)$.
10. $(x-12)(x+6)$.
11. $(x-5)(x-7)$.
12. $(x+14)(x-4)$.
13. $(x-1)(x+8)$.
14. $(x-5)(x-4)$.

15. $(x+11)(x-2)$.
16. $(x-25)(x-4)$.
17. $(x+5)(x+15)$.
18. $(x-6)(x-3)$.
19. $(x+6)(x-3)$.
20. $(x+9)(x+12)$.
21. $(x-10)(x+12)$.
22. $(x-2y)(x+4y)$.
23. $(x-a)(x-7a)$.
24. $(x+6y)(x+10y)$.

SIMULTANEOUS EQUATIONS.

66. 1. If the sum of two numbers is 8, what are the numbers? How many answers may be given to the question?

2. Let x and y stand for the two numbers; then, in the equation $x+y=8$, how many values has x? How many has y? How many values has each unknown quantity in such an equation?

3. In the equation $x+y=20$, what is the value of y, if $x=8$? If $x=6$? If $x=4$? If $x=12$? If $x=10$?

4. If the equations $x+y=6$ and $x-y=2$ are added together (Ax. 2), what is the resulting equation? What is the value of x in these equations? Of y?

67. Equations in which the same unknown quantity has the same value are called **Simultaneous Equations.**

68. The process of deducing from simultaneous equations other equations containing a less number of unknown quantities than is found in the given equations, is called **Elimination.**

69. Elimination by addition or subtraction.

1. If the equations $x+3y=9$ and $x-3y=3$ are added, what is the resulting equation? What quantity is eliminated by the addition?

2. How may the equations $2x-4y=4$ and $x+4y=8$ be combined so as to eliminate y?

3. How may the equations $3x+4y=18$ and $3x+y=9$ be combined so as to eliminate x?

4. When may a quantity be eliminated by addition? When by subtraction?

5. If $x+3y=5$ and $2x+3y=7$, how may the value of x be found?

6. If $3x-y=5$ and $2x+y=5$, how may the value of x be found?

70. Principle. ***Quantities may be eliminated by addition or subtraction when they have the same coefficients.***

71. 1. Find the value of x and y in the equations $x+3y=11$ and $2x-4y=2$.

$$
\begin{array}{rl}
x+3y=11 & (1) \\
2x-4y=2 & (2) \\ \hline
4x+12y=44 & (3) \\
6x-12y=6 & (4) \\ \hline
10x=50 & (5) \\
x=5 & (6) \\
5+3y=11 & (7) \\
3y=6 & (8) \\
y=2 & (9)
\end{array}
$$

Explanation. Since the quantities have not the same coefficients, we must multiply the equations by such numbers as will make the coefficients alike. If we wish to eliminate y, we must multiply (1) by 4 and (2) by 3 (Ax. 4). We may now eliminate y from equations (3) and (4) by addition. From the resulting equation, $10x=50$, the value of x is obtained by dividing each member by 10, the coefficient of x.

By substituting the value of x in equation (1), equation (7) is obtained, and the value of y is found to be 2.

Rule. *If necessary, multiply one or both equations by such a quantity as will cause one unknown quantity to have the same coefficient in each equation.*

When the signs of the equal coefficients are alike, subtract one equation from another; when the signs are unlike, add the equations.

Find the values of the unknown quantities in the following equations:

2. $\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$

3. $\begin{cases} x - 2y = 4 \\ 2x - y = 11 \end{cases}$

4. $\begin{cases} x + 3y = 17 \\ 2x - 2y = 2 \end{cases}$

5. $\begin{cases} x + y = 4 \\ 4x - y = 1 \end{cases}$

6. $\begin{cases} 5x - y = 8 \\ 2x + 2y = 20 \end{cases}$

7. $\begin{cases} 2x - y = 3 \\ 3x + 5y = 50 \end{cases}$

8. $\begin{cases} x + 2y = 23 \\ 3x + y = 34 \end{cases}$

9. $\begin{cases} 4x + 2y = 14 \\ 9x - y = 4 \end{cases}$

10. $\begin{cases} 4x - 7y = 17 \\ x + y = 7 \end{cases}$

11. $\begin{cases} 2x - 5y = 6 \\ 5x - 12y = 16 \end{cases}$

12. $\begin{cases} x + y = 18 \\ 7x - 2y = 9 \end{cases}$

13. $\begin{cases} 2x + 5y = 62 \\ 6x - 5y = 2 \end{cases}$

14. $\begin{cases} 5x - 3y = 4 \\ 5x + 3y = 16 \end{cases}$

15. $\begin{cases} 9x + 4y = 31 \\ 2x - 8y = -2 \end{cases}$

16. $\begin{cases} 2y + z = 26 \\ 2y + 2z = 28 \end{cases}$

17. $\begin{cases} 2y + 3z = 23 \\ 3y - 2z = 2 \end{cases}$

18. $\begin{cases} 8x - 10z = 10 \\ 3x + z = 18 \end{cases}$

19. $\begin{cases} x + 2y = 30 \\ 2x + y = 18 \end{cases}$

20. $\begin{cases} 3x - y = 30 \\ x - 3y = -30 \end{cases}$

21. $\begin{cases} 2x + 3y = 14 \\ 3x + 2y = 16 \end{cases}$

22. $\begin{cases} 10y - z = 17 \\ 14y - 3z = 3 \end{cases}$

23. $\begin{cases} 5x + 2y = 6 \\ 4x + 3y = 2 \end{cases}$

24. $\begin{cases} 6x - 3y = 12 \\ 5x - y = 13 \end{cases}$

25. $\begin{cases} 5x - 3y = 27 \\ 3x - 2y = 16 \end{cases}$

PROBLEMS.

72. 1. The sum of two numbers is 10, and their difference is 2. What are the numbers?

2. The sum of two numbers is 14, and the greater plus two times the less is 20. What are the numbers?

3. A and B together have \$ 300. Three times B's money added to five times A's gives \$ 1100. How much money has each?

4. The sum of the ages of a father and son is 50 years. The difference between the father's age and two times the son's is 20 years. What is the age of each?

5. A boy has 25 marbles in two pockets. Twice the number in one pocket equals three times the number in the other. How many marbles has he in each pocket?

6. A farmer paid \$ 3400 for 100 acres of land. For part of it he paid \$ 30 per acre, and for part of it \$ 40 per acre. How many acres were bought at each price?

7. A boy spent 35 cents for oranges and pears, buying in all 13 oranges and pears. He paid 3 cents apiece for the oranges and 2 cents apiece for the pears. How many of each kind did he buy?

8. Two men start in business with \$ 5000 capital. Twice the amount B furnishes taken from twice the amount A furnishes will leave the amount B furnishes. What capital does each furnish?

9. There are two numbers such that five times the first minus three times the second equals 4, and two times the first plus the second equals 17. What are the numbers?

10. A farmer sold 5 horses and 7 cows to one person for \$ 745. To another person at the same price per head

he sold 3 horses and 10 cows for $ 650. What was the price per head of each?

11. A man and wife working for 6 days received $ 15. Again, the man worked for 4 days and the wife 5 days, and they received $ 11. What were the daily wages of each?

12. The sum of two numbers is 26. The first minus twice the second is 8. What are the numbers?

13. A purse contained $ 30 in one and two dollar bills. If the whole number of bills was 18, how many bills were there of each kind?

14. A merchant sold 2 yards of velvet and 4 yards of broadcloth for $ 16. Again he sold 3 yards of velvet and 5 yards of broadcloth for $ 21. What was the price of each per yard?

15. If A gives B $ 5 of his money, B will have twice as much money as A has left; but if B gives A $ 5 of his money, A will have three times as much as B has left. How much money has each?

16. A boy who desired to purchase some writing pads and some pencils found that 2 pads and 7 pencils would cost him 31 cents, and that 3 pads and 4 pencils would cost 27 cents. What was the price of each?

17. The wages of 10 men and 8 boys per day were $ 28, and the wages of 7 men and 10 boys at the same rate were $ 24. What were the daily wages of each?

18. A man received at one time $ 17 for sawing 8 cords of wood and splitting 10 cords, and at another time $ 13.50 for sawing 5 cords of wood and splitting 12 cords at the same rates as on the former occasion. What did he receive per cord for the sawing and for the splitting?

DIVISION.

73. 1. Since $+5$ multiplied by $+4$ is $+20$, if $+20$ is divided by $+5$, what is the sign of the quotient?

2. What, then, is the sign of the quotient when a positive quantity is divided by a positive quantity?

3. Since $+5$ multiplied by -4 is -20, if -20 is divided by $+5$, what is the sign of the quotient?

4. What, then, is the sign of the quotient when a negative quantity is divided by a positive quantity?

5. Since -5 multiplied by $+4$ is -20, if -20 is divided by -5, what is the sign of the quotient?

6. What, then, is the sign of the quotient when a negative quantity is divided by a negative quantity?

7. Since -5 multiplied by -4 is $+20$, if $+20$ is divided by -5, what is the sign of the quotient?

8. What, then, is the sign of the quotient when a positive quantity is divided by a negative quantity?

9. What is the sign of the quotient when the dividend and the divisor have like signs? What, when they have unlike signs?

10. How many times is $6x$ contained in $12x$? $8y$ in $24y$?

11. How, then, is the coefficient of the quotient found?

12. Since $x^4 \times x^2 = x^6$, if x^6 is divided by x^2, what is the quotient? What, when x^6 is divided by x^4?

13. Since $a^5 \times a^3 = a^8$, what is the quotient if a^8 is divided by a^3? What, if a^8 is divided by a^5?

14. How, then, is the exponent of a quantity in the quotient found?

74. Division is indicated in two ways:

1. By the sign $\div$, read *divided by*.

Thus, $a \div b$ shows that a is to be divided by b.

2. By writing the dividend above the divisor with a line between them.

Thus, $\frac{a}{b}$ shows that a is to be divided by b.

75. PRINCIPLES. 1. *The sign of any term of the quotient is + when the dividend and divisor have like signs, and − when they have unlike signs.*

2. *The coefficient of the quotient is equal to the coefficient of the dividend divided by that of the divisor.*

3. *The exponent of any quantity in the quotient is equal to its exponent in the dividend diminished by its exponent in the divisor.*

76. The principle relating to the signs in division may be illustrated as follows:

$$\left.\begin{array}{l} +a \times +b = +ab \\ -a \times +b = -ab \\ +a \times -b = -ab \\ -a \times -b = +ab \end{array}\right\}. \quad \text{Hence} \left\{\begin{array}{l} +ab \div +b = +a \\ -ab \div +b = -a \\ -ab \div -b = +a \\ +ab \div -b = -a \end{array}\right.$$

77. To divide when the divisor is a monomial.

1. Divide $14x^2yz^3$ by $-7xyz$.

PROCESS.

$$-7xyz \,\big|\, \frac{14x^2yz^3}{-2xz^2}$$

EXPLANATION. Since the dividend and divisor have unlike signs, the sign of the quotient is − (Prin. 1.)

Then 14 divided by − 7 is − 2; x^2 divided by x is x (Prin. 3); y divided by y is 1, which need not appear in the quotient; z^3 divided by z is z^2. Therefore the quotient is $-2xz^2$.

2. Divide $4ax^4y^3 - 12a^2x^2y^2 - 20a^2xy^3z$ by $2axy$.

PROCESS.

$$2axy \,\big|\, \frac{4ax^4y^3 - 12a^2x^2y^2 - 20a^2xy^3z}{2x^3y^2 - 6axy - 10ay^2z}$$

EXPLANATION. When there are several terms in the dividend, each term must be divided separately.

RULE. *Divide each term of the dividend by the divisor as follows:*

Divide the coefficient of the dividend by the coefficient of the divisor. To this quotient annex each literal factor of that term of the dividend with an exponent equal to the exponent of that letter in the dividend minus its exponent in the divisor.

Write the sign + before each term of the quotient when the terms of both dividend and divisor have like signs, and − when they have unlike signs.

	3.	4.	5.	6.	7.	8.
Divide	$10a$	$16x$	$-14ab$	$15xy^2$	$-20a^2b^2$	$-18m^2n$
By	$5a$	$2x$	$7ab$	$-3xy$	$-4ab$	$6m$

Find the quotients in the following:

9. $30a^2bx^2 \div 15ax$.

10. $-24x^2y^2z^3 \div 8x^2z^2$.

11. $21ax^2y \div -7ay$.

12. $-9abc^2 \div -3abc$.

13. $30\,n^2x^2 \div 6\,n^2$.

14. $-12\,x^3y^3 \div 12\,xy$.

15. $-20\,x^2y^4 \div -10\,y^2$.

16. $-100\,x^2yz \div 25\,xyz$.

17. $80\,x^4y^2 \div 20\,xy^2$.

18. $-55\,abc^2d \div 11\,abc$.

19. $27\,x^3z^3 \div -9\,x^2z$.

20. $-120\,m^3n \div -15\,mn$.

21. $325\,x^2y^2z^4 \div 5\,xy^2z^2$.

22. $-65\,x^3z^5 \div -13\,x^2z^4$.

Divide:

23. $a^2xy - 2\,axy^2$ by ay.

24. $9\,x^2y^2 + 15\,xy^2z^2$ by $3\,xy^2$.

25. $14\,a^4b^3c + 49\,a^2bc$ by $7\,abc$.

26. $-xz^3 - 3\,xz + x^2z^2$ by $-xz$.

27. $4\,c^2d - 14\,cd^2$ by $2\,cd$.

28. $-5\,x^2y + 10\,x^2y^2 - 15\,xy^2$ by $5\,xy$.

29. $16\,m^2n^2 - 12\,m^2n - 8\,mn^2$ by $4\,mn$.

30. $15\,ax^3 - 25\,bx^2y + 35\,cxy^2$ by $-5\,x$.

31. $9\,x^2yz - 36\,xy^2z^3 + 45\,axyz^5$ by $9\,xyz$.

32. $42\,x^3 - 14\,x^2 + 28\,x + 35$ by 7.

33. $-45\,a^2b^2c^2 - 60\,abc^2 + 30\,ab^2c^3$ by $-15\,abc$.

34. $116\,m^5 + 80\,m^3 - 112\,m^2 - 92\,m$ by $4\,m$.

35. $3\,x^3yz^2 - 15\,x^5y^2z^3 + 6\,x^4yz^3 + 18\,x^6y^3z$ by $-3\,x^3yz$.

36. $3\,x^3 - 6\,x^5 + 9\,x^7 - 12\,x^9$ by $3\,x^3$.

37. $30\,x^3y^3 + 60\,x^2y^2 - 45\,xy^4 + 75\,x$ by $15\,x$.

38. $24\,abx - 16\,aby + 32\,a^2bx^2 - 8\,ab$ by $8\,ab$.

39. $-x^5y - x^4yz + xy^4z - x^3y^3z^2 + xy^2z^3$ by $-xy$.

40. $50\,x^3yz^3 + 35\,x^2y^2z - 15\,ax^2yz^3 - 20\,bxy^2z^4$ by $5\,xyz$.

41. $2\,n^2x^4y^2 - 3\,nx^3y^3 - 4\,mnx^2y^5 + 3\,n^2x^3y^2$ by nx^2y^2.

42. $a(x+y)^2 - ab(x+y)^3 + a^2b^2(x+y)^4$ by $a(x+y)^2$.

78. To divide when the divisor is a polynomial.

1. Divide $a^3 - 3a^2b + 3ab^2 - b^3$ by $a - b$.

PROCESS.

$$\begin{array}{r|l}
a^3 - 3a^2b + 3ab^2 - b^3 & a - b \\ \cline{2-2}
\underline{a^3 - a^2b} & a^2 - 2ab + b^2 \\
-2a^2b + 3ab^2 & \\
\underline{-2a^2b + 2ab^2} & \\
ab^2 - b^3 & \\
\underline{ab^2 - b^3} &
\end{array}$$

EXPLANATION. The divisor is written at the right of the dividend, and the quotient below the divisor. The first term of the divisor is contained in the first term of the dividend a^2 times. Therefore a^2 is the first term of the quotient. a^2 times the divisor, $a - b$, is $a^3 - a^2b$, and this subtracted from the partial dividend leaves a remainder of $-2a^2b$, to which the next term of the dividend is annexed for a new dividend.

The first term of the divisor is contained in the first term of the new dividend $-2ab$ times, consequently $-2ab$ is the second term of the quotient. $-2ab$ times the divisor, $a - b$, is $-2a^2b + 2ab^2$, and this subtracted from the second partial dividend leaves a remainder of ab^2, to which the next term of the dividend is annexed for a new dividend.

The first term of the divisor is contained in the first term of the new dividend b^2 times, hence b^2 is the third term of the quotient. b^2 times $a - b$, is $ab^2 - b^3$, which subtracted from the third partial dividend leaves no remainder. Hence the quotient is $a^2 - 2ab + b^2$.

RULE. *Write the divisor at the right of the dividend, arranging the terms of each according to the ascending or descending powers of one of the literal quantities.*

Divide the first term of the dividend by the first term of the divisor, and write the result for the first term of the quotient.

Multiply the divisor by this term of the quotient, subtract the product from the dividend, and to the remainder annex one or more terms for a new dividend.

Divide the new dividend as before, and continue to divide until there is no remainder, or until the first term of the divisor is not contained in the first term of the dividend.

If there is a remainder after the last division, write it over the divisor in the form of a fraction, and annex it with its proper sign to the part of the quotient previously obtained.

2.

$$
\begin{array}{r|l}
15x^2 - 8xy - 12y^2 & 3x + 2y \\
\underline{15x^2 + 10xy} \qquad\qquad & \overline{5x - 6y} \\
-18xy - 12y^2 & \\
\underline{-18xy - 12y^2} &
\end{array}
$$

3.

$$
\begin{array}{r|l}
8c^2 - 26cd + 15d^2 + 8 & 2c - 5d \\
\underline{8c^2 - 20cd} \qquad\qquad\quad & \overline{4c - 3d + \dfrac{8}{2c - 5d}} \\
-6cd + 15d^2 \quad\; & \\
\underline{-6cd + 15d^2} \quad\; & \\
+8 &
\end{array}
$$

4.

$$
\begin{array}{r|l}
a^2 + ab - ac - bc & a + b \\
\underline{a^2 + ab} \qquad\qquad & \overline{a - c} \\
-ac - bc & \\
\underline{-ac - bc} &
\end{array}
$$

5.

$$
\begin{array}{l|l}
x^3 - y^3 & x - y \\
\underline{x^3 - x^2y} & \overline{x^2 + xy + y^2} \\
\quad x^2y - y^3 & \\
\quad \underline{x^2y - xy^2} & \\
\qquad xy^2 - y^3 & \\
\qquad \underline{xy^2 - y^3} &
\end{array}
$$

Divide:

6. $x^2 + 2xy + y^2$ by $x + y$.

7. $m^2 - n^2$ by $m - n$.

8. $x^3 + 3x^2y + 3xy^2 + y^3$ by $x + y$.

9. $x^2 - 3x - 18$ by $x - 6$.

10. $x^2 + 8x + 15$ by $x + 3$.

11. $a^2 - 10ab - 24b^2$ by $a + 2b$.

12. $x^2 - 4x + 4$ by $x - 2$.

13. $x^4 - 49$ by $x^2 + 7$.

14. $a^6 - 16$ by $a^3 - 4$.

15. $10x^2 + 14x - 12$ by $2x + 4$.

16. $2x^3 - x^2 + 3x - 9$ by $2x - 3$.

17. $16a^2 - 24ab + 9b^2$ by $4a - 3b$.

18. $a^3 + b^3$ by $a + b$.

19. $x^3 - 8$ by $x - 2$.

20. $a^4 + a^3y + ay^3 + y^4$ by $a + y$.

21. $x^3 - 5x^2 - x + 14$ by $x^2 - 3x - 7$.

22. $x^3 - 86x - 140$ by $x - 10$.

23. $6x^3 + 4y^3 + 6xy^2 - 16x^2y$ by $3x^2 - y^2 - 2xy$.

24. $18x - 1 - 17x^2$ by $1 - x$.

25. $a^4 + 4a^2x^2 + 16x^4$ by $a^2 + 2ax + 4x^2$.

26. $27x^3 + 8y^3$ by $3x + 2y$.

27. $4x^5 - x^3 + 4x$ by $2 + 2x^2 + 3x$.

28. $x^4 - 9x^2 - 6xy - y^2$ by $x^2 + 3x + y$.

29. $36 + x^4 - 13x^2$ by $6 + x^2 + 5x$.

30. $1 - x - 3x^2 - x^5$ by $1 + 2x + x^2$.

PROBLEMS.

79. 1. Express the sum of x and y divided by 2.

2. If x oranges are worth 10 cents, how much is one orange worth?

3. How many apples at 2 cents apiece, can be bought for m cents?

4. Express the quotient of b divided by a.

5. If 5 bananas cost c cents, what will 2 bananas cost?

6. How many days will it take a man to earn \$15, if he receives a dollars a day?

7. A farmer has a cows, $\frac{1}{2}$ as many horses, and three times as many sheep as horses. How many animals has he altogether?

8. Henry and James together had d marbles; Henry had c times as many as James. What represents the number that each had?

9. What is the cost per barrel, if b barrels of flour cost the sum of m and n dollars?

10. A, B, and C start in business. A furnishes $2a$ dollars, which is c times as much as B furnishes; and C furnishes b times as much as B. How much do B and C furnish?

11. A grocer mixed together equal quantities of three kinds of coffee, worth a cents, b cents, and c cents per pound. What is the cost of one pound of the mixture?

GENERAL REVIEW EXERCISES.

80. 1. Add $6(x+y)+3z-8$, $2(x+y)-2z+4$, $8z-3(x+y)$.

2. Add a^2+b^2+4ab, $5a^2-5b^2-5ab$, $-3a^2+2b^2-4ab$.

3. Add $\frac{1}{2}x+\frac{1}{3}y+z$, $x+\frac{2}{3}y+\frac{1}{3}z$, $\frac{1}{2}x+y+\frac{2}{3}z$.

4. Add $2x^m-5xy^n+6$, $3xy^n+4y^n-5x^m$, $6x^m-10+7xy^n$.

5. Add $\frac{3}{5}(m-n)+\frac{1}{5}(m+n)$, $\frac{2}{5}(m-n)-\frac{3}{5}(m+n)$, $\frac{1}{5}(m-n)-\frac{2}{5}(m+n)$.

6. From $9x+9x^2-3x^3$ subtract $2x-2x^2-12x^3$.

7. From $\frac{3}{4}a-\frac{1}{4}b+\frac{1}{6}c$ subtract $\frac{1}{2}a+\frac{1}{2}b-\frac{5}{6}c$.

8. From $5x^my-7x^2y^2+15x^mz^n$ subtract $2x^2y^2-3x^my+5x^mz^n$.

9. From $3(x+y)^2-2z^2+8$ subtract $2z^2-x^2+4(x+y)^2$.

10. From $3a^2-2a^2x-7$ subtract $7+3a^2-a^2x+2m$.

11. Multiply $m^4-m^3+m^2-m+1$ by $m+1$.

12. Multiply $a+b+c+e$ by $a+b+c+e$.

13. Multiply $x^4+2x^3y+4x^2y^2+8xy^3+16y^4$ by $x-2y$.

14. Find the product of $x-10$, $x+4$, and $x+6$.

15. Find the product of $a-b$, $a+b$, and a^2-b^2.

16. Find the product of $2a-5$, $2a-5$, $2a-5$, and $2a+5$.

17. Divide $4a^4-5a^2b^2+b^4$ by $2a^2-3ab+b^2$.

18. Divide a^5+1 by $a+1$.

19. Divide $m^2-c^2+2cz-z^2$ by $m+c-z$.

20. Divide $1-x-3x^2-x^5$ by $1+2x+x^2$.

21. Divide $6x^4-96$ by $3x-6$.

Find the value of x in the following:

22. $10x-3x+4=x+10-2x+8+x$.

23. $6x-13-9x+x=4x-12+3x-6x-13$.

24. $5(x+1)+6(x+2)=6(x+7)$.

25. $3(x+1)+4(x+2)=6(x+3)$.

26. $\frac{1}{2}x-4+\frac{3}{4}x=16+\frac{1}{4}x-10$.

27. $x(x+5)-6=x(x-1)+12$.

28. $3(2-x)-2(x+3)=6-2x$.

29. $x-(2+4x)=13-5(x+5)$.

30. $2(3x-2)-5(x-1)=5-x$.

31. A man whose annual income is c dollars owes $a+b$ dollars. In how many years can he pay his debts, if his annual expenses are d dollars?

32. A man being asked how much money he had, replied that \$25 more than three times what he had would equal \$775. How much money had he?

33. A man drove 155 miles in three days. On the second day he drove 15 miles farther than on the first, and on the third day he drove 20 miles farther than on the first. How far did he drive each day?

34. A and B start from two towns 231 miles apart and travel toward each other. A goes 15 miles per day, and B goes 18 miles per day. In how many days will they meet?

35. A man worked 20 days during a certain month. A part of the time he received \$1 per day, and part of the time \$1½ per day. If he received \$25 for his wages, how many days did he work at \$1, how many at \$1½ per day?

36. David and his father earned \$100 during a certain month. David earned \$10 more than ½ as much as his father. How much did each earn?

37. A man who paid ⅛ of his wages for board found that in 24 days he saved \$27. If his other expenses during that time were \$5, what were his daily wages?

38. A man bought an overcoat, a suit of clothes, and a pair of boots for \$49. If the overcoat cost \$4 less than the suit of clothes, and \$15 more than the boots, what was the cost of each?

39. A house and two lots cost \$4000. If one lot cost \$250, and the other ¼ as much as the house, how much did the other lot cost?

40. A fruit grower realized \$35 from the sale of three varieties of apples, receiving for them, respectively, 60 cents, 90 cents, and \$1 a bushel. If he sold the same number of bushels of each of the three kinds, how many bushels of each did he sell?

FACTORING.

81. The quantities which, when multiplied together, produce a quantity, are called **Factors** of the quantity.

Thus, a, b, and $(x + y)$ are the factors of $ab(x + y)$.

a, b, and $(x + y)$ are called the *prime factors* of $ab(x + y)$, because they have no factors besides themselves and 1.

82. A factor of a quantity is an **Exact Divisor** of it.

83. The process of separating a quantity into its factors is called **Factoring.**

84. To factor a polynomial when all the terms have a common factor.

1. What are the factors of $4a^2m - 6am^2x + 10a^2m^2x^2$?

PROCESS.

$$2am \,\underline{\big|\, 4a^2m - 6am^2x + 10a^2m^2x^2}$$
$$ 2a - 3mx + 5amx^2$$

Explanation. By examining the terms of a polynomial, we find that $2am$ is the highest factor common to all the terms. Dividing by $2am$, we obtain the other factor. Hence, the two factors are $2am$ and $2a - 3mx + 5amx^2$.

The same result may be secured by separating the terms of the polynomial into their prime factors and then selecting the *common factors.*

Rule. *Divide the polynomial by the highest factor or divisor common to all the terms. The divisor and quotient will be the factors sought.*

Find the factors of the following polynomials:

2. $18x^2 - 27xy$.
3. $15x^3y^2 + 20x^4y$.
4. $12m^2n^2 - 48mn^3$.
5. $5abc - 5ac^2 + 15ab^2c$.
6. $6x^2 + 4xy - 8x^3$.
7. $9x^2y^2 - 6x^2y + 12x^2yz$.
8. $6a^4b + 21a^2b - 18a^3b^3$.
9. $3a^2b - abc - abd$.
10. $6a^4b^5 + 36a^3b^2 - 42a^3b^3$.
11. $15a^2b^4 + 20a^3b^5 - 25a^5b^2$.
12. $20m^3 - 50m^2 + 30m^4$.
13. $xy^2 + 9x^2y^2 + 27x^3y^3$.
14. $5am^2 + 10mn + 15mn^2$.
15. $45x^2y^2z + 60x^2yz^2$.
16. $39x^2y^4 - 65x^3y + 91x^2y^3$.
17. $32a^3b^6 + 96a^6b^8 - 8a^8b^9$.
18. $25an + 75am^2 - 15an^2$.
19. $15ax^3 - 35x^3y^3 - 55bx^3$.
20. $45b^2x^4 + 60x^4y - 30x^4z$.
21. $72a^3y^4 - 36a^2y^3 - 108a^2y^5$.

85. To factor a polynomial when some of the terms have a common factor.

1. Factor $3xy + 3xz + ay + az$.

SOLUTION.
$$\begin{aligned} &3xy + 3xz + ay + az \\ &= 3x(y+z) + a(y+z) \\ &= (3x+a)(y+z) \end{aligned}$$

Factor the following:

2. $ax + by + bx + ay$.
3. $ab + 2b + 3a + 6$.
4. $9 + 3x + 3y + xy$.
5. $ax + ay - bx - by$.
6. $xy + 6 - bxy - 6b$.
7. $y^3 - y^2 + y - 1$.
8. $x^3 + x^2y - xy^2 - y^3$.
9. $x^2y^2 + bx^2 - b^2y^2 - b^3$.
10. $2a^4 - a^3 + 4a - 2$.
11. $ax - ab - bx + b^2$.
12. $x^3 + x^2 + ax + a$.
13. $ax^2 + ay^2 - bx^2 - by^2$.
14. $1 + a^2 - a^3 - a^5$.
15. $1 - x - x^2 + x^3$.
16. $x^2y - x^2z - xy^2 + xyz$.
17. $a + ay + ay^2 + ay^3$.

86. To separate a trinomial into two equal factors.

$$(m+n)(m+n)=m^2+2mn+n^2.$$
$$(m-n)(m-n)=m^2-2mn+n^2.$$

1. What is the product of $(m+n)(m+n)$? What, then, are the factors of $m^2+2mn+n^2$? How are the terms of the factors found from the trinomial?

2. What is the product of $(m-n)(m-n)$? What, then, are the factors of $m^2-2mn+n^2$? How are the terms of the factors found from the trinomial?

3. What term of the trinomial determines the sign which connects the terms of the binomial factors?

87. One of the *two equal* factors of a quantity is called its **square root.**

RULE. *Find the square roots of the terms that are squares and connect these roots by the sign of the other term. The result will be one of the equal factors.*

The other term must always be twice the product of the square roots of the terms that are squares.

Find the equal factors of the following trinomials:

1. $x^2+2xy+y^2$
2. $x^2+4x+4.$
3. $4a^2-4ab+b^2.$
4. $9m^2+6mn+n^2.$
5. $x^2-4xy+4y^2.$
6. $y^2+2y+1.$
7. $a^2b^2-8ab+16.$
8. $4n^2-20n+25.$
9. $4x^2+8xy+4y^2.$
10. $x^2-10x+25.$
11. $1+2z+z^2.$
12. $a^4+4a^2b^2+4b^4.$
13. $x^2-6xz+9z^2.$
14. $x^2+20x+100.$
15. $m^2+8mn+16n^2.$
16. $9x^2-18xy+9y^2.$

17. $9+6a^2+a^4$.

18. $4a^4x^2-4a^2b^2x+b^4$.

19. $a^2m^2-8amn+16n^2$.

20. $25x^4+80x^2+64$.

21. $a^4+4a^2x^2+4x^4$.

22. $4b^2c^2-12bcd+9d^2$.

23. a^4+18a^2+81.

24. $100x^2-20x+1$.

25. $m^2n^2+4mn+4$.

26. $x^{2n}+2x^ny^n+y^{2n}$.

27. $16+16m^2+4m^4$.

28. $1-8x^m+16x^{2m}$.

88. To resolve a binomial into two binomial factors.

$$(a+b)(a-b)=a^2-b^2.$$

1. What is the product of $(a+b)(a-b)$? What, then, are the factors of a^2-b^2? How do these two factors differ?

2. What is the product of $(x+3)(x-3)$? What, then, are the factors of x^2-9?

RULE. *Find the square root of each term of the binomial and make the sum of these square roots one factor and their difference the other.*

A binomial cannot be factored by the above rule unless the second term is *negative* and the indices of the powers are *even numbers.*

Sometimes the factors of such a quantity may themselves be resolved into factors.

Thus,
$$\begin{aligned} x^4-y^4 &= (x^2+y^2)(x^2-y^2) \\ &= (x^2+y^2)(x+y)(x-y). \end{aligned}$$

Factor the following:

1. x^2-y^2

2. x^4-4.

3. a^2-9b^2.

4. m^2-1.

5. $a^2b^2-c^2$.

6. x^2-25.

7. $121x^2-100y^2$.

8. $16a^2-9b^2$.

9. x^4-144.

10. $1-z^2$.

11. $x^4 - y^4$.

12. $x^2 - 16$.

13. $9x^2 - 36$.

14. $4x^2 - 25$.

15. $x^4 - 81$.

16. $m^2n^2 - 64x^2$.

17. $4a^2 - 16b^2$.

18. $x^4 - 625$.

19. $9x^2 - 81$.

20. $25a^2b^2 - 49c^4d^4$.

21. $m^4 - 625$.

22. $225x^2 - 100y^2$.

23. $16y^4 - 256$.

24. $a^2c^2 - 9d^2$.

25. $x^2 - 121$.

26. $c^2d^2 - 1$.

27. $25m^2 - 225n^2$.

28. $x^6 - 1$.

29. $x^{2m} - y^{2n}$.

30. $x^{8m} - 169$.

89. To factor a quadratic trinomial.

$$(x+3)(x+2) = x^2 + 5x + 6.$$
$$(x+3)(x-2) = x^2 + x - 6.$$
$$(x-3)(x+2) = x^2 - x - 6.$$
$$(x-3)(x-2) = x^2 - 5x + 6.$$

1. In the above examples, what terms of the product are alike?

2. How may the first term of each factor be found from the product? How may the second term of each factor be found from the product?

3. How is the coefficient of the second term in the products found from the last terms of the factors? How, then, may the sign of the second term of each factor be found from the second term of the product?

90. A trinomial in the form, $x^2 \pm ax \pm b$, in which b is the product of two quantities, and a their algebraic sum, is called a **Quadratic Trinomial.**

1. Resolve $x^2 - 9x + 18$ into two factors.

PROCESS.

$$x^2 - 9x + 18$$

$$18 = \begin{cases} 18 \times 1 \\ 9 \times 2 \\ 6 \times 3 \end{cases}$$

$$-9 = -6 - 3$$

$$(x-6)(x-3)$$

EXPLANATION. The first term of each factor is evidently x. Since 18 is the product of the other two quantities, and -9 is their sum, we must select from the different pairs of factors of 18 those two whose sum is -9. Therefore, -6 and -3 are the other terms. Hence, $x-6$ and $x-3$ are the factors required.

2. Factor $x^2 + 4x - 21$.

SOLUTION. $21 = 3 \times 7$, or 21×1.
The factors whose algebraic sum is $+4$ are $+7$ and -3.
$\therefore$ $(x+7)$ and $(x-3)$ are the factors.

Factor the following:

3. $a^2 + 6a + 8$.
4. $x^2 + 5x - 14$.
5. $x^2 - 6x - 7$.
6. $m^2 - 10m + 9$.
7. $b^2 + 5b + 6$.
8. $n^2 - n - 2$.
9. $n^2 + n - 2$.
10. $x^2 - 7x - 18$.
11. $x^2 + 4x - 12$.
12. $a^2 - 5a - 24$.
13. $x^2 + 5x - 36$.
14. $y^2 - 3yz + 2z^2$.
15. $m^2 - 6mn - 16n^2$.
16. $a^2 + 9ax + 8x^2$.
17. $x^2 - 10x - 200$.
18. $a^2 - 6ax - 55x^2$.
19. $x^2 - 14x + 40$.
20. $x^2 + xy - 20y^2$.
21. $m^2 + 10m + 24$.
22. $a^2 - 4ab - 21b^2$.
23. $x^2 - 14x + 48$.
24. $x^2 + 7x - 18$.
25. $c^2 + c - 420$.
26. $a^2 + 11ab + 28b^2$.
27. $x^{2n} - 9x^n + 20$.
28. $x^4 - 3x^2 - 154$.
29. $m^2 + 2m - 255$.
30. $x^{2c} + 12x^c + 35$.

91. To factor the sum or the difference of two cubes.

$$(a^3+b^3)\div(a+b)=a^2-ab+b^2.$$
$$(a^3-b^3)\div(a-b)=a^2+ab+b^2.$$

92. PRINCIPLE. *The sum of the cubes of two quantities is divisible by the sum of those quantities, and the difference of the cubes of two quantities is divisible by the difference of those quantities.*

1. Factor a^3-x^3.

SOLUTION. a^3-x^3 is divisible by $a-x$ (Prin.)

$$(a^3-x^3)\div(a-x)=a^2+ax+x^2$$
$$\therefore\ a^3-x^3=(a-x)(a^2+ax+x^2)$$

2. Factor r^3+s^3.

SOLUTION. r^3+s^3 is divisible by $r+s$ (Prin.)

$$(r^3+s^3)\div(r+s)=r^2-rs+s^2$$
$$\therefore\ r^3+s^3=(r+s)(r^2-rs+s^2)$$

3. Factor $a^3-b^3c^3$.

SOLUTION. $a^3-b^3c^3$ is divisible by $a-bc$

$$(a^3-b^3c^3)\div(a-bc)=a^2+abc+b^2c^2$$
$$\therefore\ a^3-b^3c^3=(a-bc)(a^2+abc+b^2c^2)$$

Observe carefully, in the examples solved above, the quantities in the quotients and the signs, and you will be able to *write out* the factors.

Factor the following:

4. x^3+y^3.	10. s^3+t^3.	16. $x^3-y^3z^3$.
5. a^3+b^3.	11. c^3-d^3.	17. $a^3+b^3c^3$.
6. m^3-n^3.	12. y^3+1.	18. $m^3n^3+c^3d^3$.
7. a^3-b^3.	13. x^3-1.	19. $a^3x^3-b^3y^3$.
8. m^3+n^3.	14. $a^3-c^3d^3$.	20. x^3+1.
9. x^3-y^3.	15. m^3+s^3.	21. y^3-1.

EQUATIONS SOLVED BY FACTORING.

93. 1. Find the value of x in the equation $x^2+1=5$.

PROCESS.

$$x^2+1=5$$
$$x^2=5-1$$
$$x^2=4$$
$$x\cdot x=2\cdot 2 \text{ or } -2\cdot -2$$
$$\therefore x=\pm 2$$

EXPLANATION. Transposing the known quantity to the second member, the first member contains the second power, only, of the unknown quantity. Separating each member into two equal factors, the equation becomes $x\cdot x=2\cdot 2$ or $-2\cdot -2$. Since each member is composed of two equal factors, a factor in each must be equal. Hence, $x=+2$ or -2; or $x=\pm 2$.

The sign, $\pm$, called the **Ambiguous Sign,** is a combination of the signs of addition and subtraction.

Thus, $a\pm b$ indicates that b may be added to or subtracted from a.

2. Find the value of x in the equation $x^2+5=30+11$.

SOLUTION.

$$x^2+5=30+11$$
$$x^2=30+11-5$$
$$x^2=36$$
$$x\cdot x=6\cdot 6 \text{ or } -6\cdot -6$$
$$\therefore x=\pm 6$$

Find the value of x in the following equations:

3. $x^2-4=5$.
4. $x^2-9=16$.
5. $x^2-25=24$.
6. $x^2-1=3$.
7. $x^2+4=20$.
8. $x^2+5=41$.
9. $x^2+2=11$.
10. $x^2-8=8$.
11. $x^2+7=11$.
12. $x^2+21=25$.
13. $x^2+1=82$.
14. $x^2+12=48$.
15. $x^2-15=10$.
16. $x^2-10=90$.
17. $x^2+15-8=8$.
18. $x^2+3-6=5-2+10$.
19. $x^2-4a^2=21a^2$.
20. $x^2+3c^2=7c^2$.

94. 21. Find the value of x in the equation $x^2+4x+4=16$.

PROCESS.

$$x^2+4x+4=16$$
$$(x+2)(x+2)=4\cdot 4 \text{ or } -4\cdot -4$$
$$\therefore\ x+2=4 \text{ or } -4$$
and $$x=2 \text{ or } -6$$

EXPLANATION. Since each member of the equation can be resolved into *two equal* factors, one factor of the first member must be equal to one factor of the second member. Hence, $x+2=4$ or -4, and x is found to have two values, 2 or -6.

22. Find the value of x in the equation $x^2+6x+9=49$.

SOLUTION.
$$x^2+6x+9=49$$
$$(x+3)(x+3)=7\cdot 7 \text{ or } -7\cdot -7$$
$$\therefore\ x+3=7 \text{ or } -7$$
and $$x=4 \text{ or } -10$$

Observe that the first member cannot be separated into two equal factors except when the trinomial is a perfect square (Art. 86).

Find the values of the unknown quantities in the following equations:

23. $y^2-10y+25=16$.
24. $x^2-8x+16=81$.
25. $x^2-16x+64=9$.
26. $z^2+12z+36=64$.
27. $x^2+14x+49=100$.
28. $x^2-20x+100=25$.
29. $y^2-18y+81=16$.
30. $x^2+22x+121=144$.
31. $x^2+6x+9=4$.
32. $x^2+24x+144=225$.
33. $x^2+2x+1=36$.
34. $y^2-14y+49=9$.
35. $z^2+30z+225=625$.
36. $x^2+24x+144=169$.
37. $x^2+40x+400=900$.
38. $x^2-26x+169=196$.
39. $y^2-2y+1=25$.
40. $x^2+50x+625=1600$.

95. **41.** Find the value of x in the equation $x^2+4x=-4$.

PROCESS.

$$x^2+4x=-4$$
$$x^2+4x+4=0$$
$$(x+2)(x+2)=0$$
$$\therefore\ x+2=0$$
and $$x=-2$$

EXPLANATION. By transposing -4 to the first member, that member becomes a perfect square which may be resolved into *two equal* factors, $(x+2)$ $(x+2)$. Since the product of these factors is 0, one of the factors must be 0; and since both factors are the same, each factor is equal to 0. Hence, the value of x is -2.

42. Find the value of x in the equation $x^2-8x=-16$.

SOLUTION.

$$x^2-8x+16=0$$
$$(x-4)(x-4)=0$$
$$\therefore\ x-4=0$$
and $$x=4$$

Find the values of x in the following equations:

43. $x^2+10x+25=0$.

44. $x^2+12x+36=0$.

45. $x^2+6x+9=0$.

46. $x^2-18x+81=0$.

47. $x^2+16x+64=0$.

48. $x^2+26x+169=0$.

49. $x^2-14x+49=0$.

50. $x^2+30x+225=0$.

51. $x^2-28x+196=0$.

52. $x^2+20x+100=0$.

53. $x^2+24x=-144$.

54. $x^2+36x=-324$.

55. $x^2-50x=-625$.

56. $x^2+22x=-121$.

57. $x^2-100x=-2500$.

58. $x^2+44x=-484$.

59. $x^2+60x=-900$.

60. $x^2+80x=-1600$.

96. **61.** Find the value of x in the equation $x^2+5x-14=0$.

PROCESS.

$$x^2+5x-14=0$$
$$(x+7)(x-2)=0$$
$$\therefore\ x+7=0,\text{ or } x-2=0$$
and $$x=-7,\text{ or } x=2$$

EXPLANATION. Factoring the first member of the equation, the factors are $(x+7)(x-2)$. Since the product of the factors is equal to 0, one of the factors is equal to 0. Solving, the values of x are found to be -7 or 2.

62. Find the value of x in the equation $x^2+x-72=0$.

SOLUTION.
$$x^2+x-72=0$$
$$(x+9)(x-8)=0$$
$$\therefore\ x+9=0, \text{ or } x-8=0$$
and
$$x=-9, \text{ or } x=8$$

Find the values of x in the following equations:

63. $x^2-2x-15=0$.
64. $x^2+6x+5=0$.
65. $x^2+10x+9=0$.
66. $x^2-8x+16=0$.
67. $x^2+2x-48=0$.
68. $x^2+13x+40=0$.
69. $x^2-5x-24=0$.
70. $x^2+7x+12=0$.
71. $x^2+9x-22=0$.
72. $x^2+15x+50=0$.
73. $x^2-x-2=0$.
74. $x^2-18x+77=0$.
75. $x^2+2x-120=0$.
76. $x^2-22x-75=0$.
77. $x^2-6ax+8a^2=0$.
78. $x^2+3x-54=0$.
79. $x^2-4ax-96a^2=0$.
80. $x^2+11bx+24b^2=0$.

97. 81. Find the value of x in the equation $x^2+6x+7=23$.

PROCESS.

$$x^2+6x+7=23$$
$$x^2+6x+7+2=23+2$$
$$x^2+6x+9=25$$
$$(x+3)(x+3)=5\cdot 5 \text{ or } -5\cdot -5$$
$$\therefore\ x+3=5 \text{ or } -5$$
and
$$x=2 \text{ or } -8$$

EXPLANATION. Equations like this may be solved in the same manner as the equations immediately preceding, by transposing all the quantities to the first member and then factoring; or they may be solved by making the first member a *perfect square* by adding to or subtracting from both members some number. The first member of

the equation is a trinomial. A trinomial is a perfect square when it is composed of two terms that are perfect squares and when the other term is twice the product of the square roots of the terms that are squares. $x^2 + 6x$ are two terms of the trinomial which is to be made a square, but the third term is to be found. Since the second term, $6x$, is twice the product of the square roots of the terms that are squares, and the square root of one of the terms is x, if $6x$ is divided by $2x$, the square root of the other term that is a square will be found. Dividing, the quotient is 3, and 3^2, or 9, is the third term of the trinomial. Since the given term is 7, 2 must be added to both members to make the first member a perfect square, giving $x^2+6x+9=25$. Factoring, $(x+3)(x+3) = 5 \cdot 5$ or $-5 \cdot -5$. Whence, $x = 2$ or -8.

82. Find the value of x in the equation $x^2-12x+33=46$.

SOLUTION.
$$x^2 - 12x + 33 = 46$$
$$x^2 - 12x + 33 + 3 = 46 + 3$$
$$x^2 - 12x + 36 = 49$$
$$(x-6)(x-6) = 7 \cdot 7 \text{ or } -7 \cdot -7$$
$$\therefore x - 6 = 7 \text{ or } -7$$
and
$$x = 13 \text{ or } -1$$

Solve the following equations:

83. $x^2 + 10x + 20 = 11$.

84. $x^2 + 8x + 12 = 32$.

85. $x^2 - 18x + 80 = 15$.

86. $x^2 + 4x + 2 = 7$.

87. $x^2 - 20x + 85 = 10$.

88. $x^2 + 14x + 45 = 60$.

89. $x^2 + 22x + 100 = 60$.

90. $x^2 + 4x + 1 = 33$.

91. $y^2 + 16y + 54 = 90$.

92. $x^2 - 24x + 122 = 3$.

93. $x^2 - 30x + 220 = 76$.

94. $x^2 + 40x + 200 = 425$.

95. $x^2 - 8x + 15 = 99$.

96. $x^2 + 12x + 27 = 40$.

97. $x^2 - 38x + 360 = 8$.

98. $y^2 + 2y - 1 = 2$.

99. $x^2 - 6x - 3 = 13$.

100. $x^2 + 8x - 2 = 18$.

COMMON DIVISORS OR FACTORS.

98. 1. Name a common divisor or factor of $5x$ and $10xy$. Of $4ab$ and $16ab$.

2. Name all the common divisors or factors of $24x^2y^2$ and $12x^3y$. Which of these is the highest common divisor or factor? Name all the common divisors or factors of $15a^2b^2$ and $25ab^4$. Which is the highest common divisor or factor?

3. What prime factors or divisors are common to $24x^2y^2$ and $12x^3y$? To $15a^2b^2$ and $25ab^4$?

4. How may the highest common divisor or factor be obtained from the prime factors of $24x^2y^2$ and $12x^3y$? How from the prime factors of $15a^2b^2$ and $25ab^4$?

99. An exact divisor or factor of two or more quantities is called a **Common Divisor** or **Factor** of both of these quantities.

Thus, $3a$ is a common divisor or factor of $9a^2$ and $12a$.

100. The divisor or factor of the highest degree that is an exact divisor of two or more quantities is called the **Highest Common Divisor or Factor.**

Thus, $5a^2x$ is the highest common divisor of $20a^3x$ and $15a^2x^3$.

101. PRINCIPLE. *The highest common divisor or factor of two or more quantities is equal to the product of all their common prime factors.*

102. To find the highest common divisor or factor of quantities that can be factored readily.

1. What is the highest common divisor of $4a^2b$ and $12a^2b^2c$?

PROCESS.

$$\begin{array}{r l} 4a^2b & = 2\cdot 2\cdot a\cdot a\cdot b \\ 12a^2b^2c & = 3\cdot 2\cdot 2\cdot a\cdot a\cdot b\cdot b\cdot c \\ \hline \text{H. C. D.} & = 2\cdot 2\cdot a\cdot a\cdot b = 4a^2b \end{array}$$

EXPLANATION. Since the highest common divisor is the product of all the common prime factors (Prin.), the quantities are separated into their prime factors. The only prime factors common to the given quantities are 2, 2, a, a, b; and their product is $4a^2b$. Therefore the highest common divisor is $4a^2b$.

2. What is the highest common divisor of $m(a^2-b^2)$ and $m(a^2-2ab+b^2)$?

SOLUTION.

$$\begin{array}{r l} m(a^2-b^2) & = m(a+b)(a-b) \\ m(a^2-2ab+b^2) & = m(a-b)(a-b) \\ \hline \text{H. C. D.} & = m\times(a-b) = m(a-b) \end{array}$$

Find the highest common divisor or factor of the following:

3. $14x^2yz^2$ and $21xy^2z$.

4. $18xyz^2$ and $45xyz$.

5. $5a^2b^2$, $30ab^2c$, and $15a^2bc$.

6. $22m^2n^3z$, $44m^2n^2z^2$, and $121m^2n^4z^2$.

7. $20abx^4$, $40ab^2x^3$, and $120a^2bx^2$.

8. $9a^2bm^3n^3$, $27b^4m^3n$, and $81bm^2n^2$.

9. x^2-1 and x^2-2x+1.

10. $x^2+2xy+y^2$ and x^2-y^2.

11. m^3+n^3 and m^2-n^2.

12. $a - b$, $a^2 - b^2$, and $a^2 - 2ab + b^2$.

13. $x^2 - 1$ and $x^2 - x - 2$.

14. $x^2 - 2x$ and $2xy^2 - 4y^3$.

15. $yz - z$ and $y^2 - 1$.

16. $1 - a^2$ and $1 + a^3$.

17. $x^2 + 2x - 3$ and $x^2 + 5x + 6$.

18. $x^2 - 2x - 15$ and $x^2 + 2x - 3$.

19. $x^2 - 3x - 4$ and $x^2 - x - 12$.

20. $x^2 - 1$, $x^4 - 1$, and $x^4 - 2x^2 + 1$.

21. $x^2 - 7x + 6$ and $x^2 + 3x - 4$.

22. $ax + bx$, $a^2m - b^2m$, and $a^2 + 2ab + b^2$.

23. $x^2 + 2x - 35$ and $x^2 + x - 42$.

24. $x^2 - 4xy + 4y^2$ and $x^2 - 4y^2$.

25. $x^2 - 8x + 15$, $x^2 - 4x - 5$, and $x^2 - 3x - 10$.

26. $x^2 + x - 20$, $x^2 - x - 12$, and $x^2 - 2x - 8$.

27. $x^2 + 2xy - 8y^2$ and $x^2 - 5xy + 6y^2$.

28. $x^2 + 4xy - 21y^2$ and $x^2 + 6xy - 7y^2$.

29. $x^2 + 6xy - 7y^2$ and $x^2 - 2xy + y^2$.

30. $x^2 - 4x - 5$ and $x^2 + 2x - 35$.

31. $x^2 - 4y^2$, $x^2 + 4xy - 12y^2$, and $x^2 - 4xy + 4y^2$.

32. $ax - 3a$, $x^2 - 7x + 12$, and $ax^2 + 5ax - 24a$.

33. $am + 2mx$, $a^2 + 4ax + 4x^2$, and $a^2 - 2ax - 8x^2$.

34. $b^2 - c^2$, $b^2 + 5bc + 4c^2$, and $b^2 - 9bc - 10c^2$.

35. $x^2 - 2x + 1$, $x^2 - 8x + 7$, and $x^2 - 4x + 3$.

36. $a^2 - 9$, $a^2 - 9a - 36$, and $a^2 - 7a - 30$.

37. $4a - 8y$, $a^2 - 5ay + 6y^2$, and $am - 2my$.

38. $2x + 6y$, $2(x^2 + 6xy + 9y^2)$, and $2ax + 6ay$.

COMMON MULTIPLES.

103. 1. What quantities will exactly contain 3, 5, x, and y?

2. What quantities will exactly contain $3x^2y^2$ and $9x^3y$?

3. Give several quantities which will exactly contain $6x^2y^3z$ and $4x^2yz$. Which one of them is the lowest quantity?

4. What factors of the given quantities must be contained in the lowest quantity?

5. If any quantity is used several times as a factor in some of the given quantities, how many times will it be employed in the lowest quantity that contains the given quantities?

104. A quantity that will exactly contain a quantity is called a **Multiple** of the quantity.

Thus, a^3x is a multiple of a, a^2, a^3, x, ax, and a^2x.

105. The lowest quantity that will exactly contain each of two or more quantities is called the **Lowest Common Multiple** of the quantities.

Thus, $4x^2y$ is the lowest common multiple of $4x$, $2y$, xy, and x^2.

106. PRINCIPLE. *The lowest common multiple of two or more quantities is equal to the product of all their different prime factors, using each factor the greatest number of times it is found in any of the given quantities.*

107. 1. What is the lowest common multiple of $12x^2yz^4$, $6a^2xy^2$, and $8axyz^2$?

PROCESS.

$$\begin{aligned} 12x^2yz^4 &= 4 \cdot 3 \cdot x^2 \cdot y \cdot z^4 \\ 6a^2xy^2 &= 6 \cdot a^2 \cdot x \cdot y^2 \\ 8axyz^2 &= 4 \cdot 2 \cdot a \cdot x \cdot y \cdot z^2 \\ \hline \text{L. C. M.} &= 4 \cdot 3 \cdot 2 \cdot a^2 \cdot x^2 \cdot y^2 \cdot z^4 = 24a^2x^2y^2z^4. \end{aligned}$$

EXPLANATION. Since the lowest common multiple is equal to the product of all the different prime factors of the given quantities (Prin.), the quantities are separated into their prime factors, when necessary.

Since any number which will contain 12 will contain 6, it is not necessary to factor 6; and since 12 and 8 contain the common factor 4 and other factors, 3 and 2, which are prime numbers, 4, 3, and 2 are the numerical factors of the L. C. M. These multiplied by all the different literal factors, each with its highest exponent showing the greatest number of times it is used as a factor in any of the given quantities, will be the lowest common multiple.

2. What is the lowest common multiple of $x^2 - 3x - 40$ and $x^2 + 3x - 10$?

SOLUTION.

$$\begin{aligned} x^2 - 3x - 40 &= (x-8)(x+5) \\ x^2 + 3x - 10 &= (x-2)(x+5) \\ \hline \text{L. C. M.} &= (x-8)(x+5)(x-2) \\ &= x^3 - 5x^2 - 34x + 80 \end{aligned}$$

Find the lowest common multiple of the following:

3. $8a^2b^2c$ and $18a^3bc^2$.

4. $10ab^2x$ and $15a^2bx^2$.

5. $14a^2b^2c^2$, $7b^2x^2y$, and $35abcx$.

6. $27am$, $33a^2my$, and $81a^2m^2y^2$.

7. $15x^3y^2z$ and $21x^2y^3z^2$.

8. $x^2 - 4$ and $x^2 - 4x + 4$.

9. $x^2 - y^2$ and $x^2 + 2xy + y^2$.

10. $a^2(x - y)$ and $a(x^2 - y^2)$.

11. $a^2 - b^2$, $a^2 + b^2$, and $a^4 - b^4$.

12. $x^2 - 9x - 22$ and $x^2 - 13x + 22$.

13. $x^2 + 5x + 6$ and $x^2 + 6x + 8$.

14. $x^4 - 16$, $x^2 + 4x + 4$, and $x^2 - 4$.

15. $c^2 - 5c + 4$ and $c^2 - 8c + 16$.

16. $m(a + b)$, $m^2(a - b)$, and $mx(a^2 - b^2)$.

17. $x(a^3 - b^3)$, $x(a - b)$, and $x^2(a^2 + ab + b^2)$.

18. $2a + 1$, $4a^2 - 1$, and $8a^3 + 1$.

19. $a^2 - a - 20$ and $a^2 + a - 12$.

20. $x^2y - xy^2$, $x^2 - y^2$, and $x^2 + xy$.

21. $x^2 - x$, $x^3 - 1$, $xy - y$, and $ab(x^3 - 1)$.

22. $x^4 - a^4$, $x^2 - a^2$, and $x - a$.

23. $a^2 - 5ab + 4b^2$ and $a^2 - 2ab + b^2$.

24. $x^2 - x - 30$ and $x^2 + 11x + 30$.

25. $x^2 - 1$ and $x^3 + 1$.

26. $x^2 - 1$ and $x^2 + 4x + 3$.

27. $b(a + b)$, b^2, $am(a - b)$, and $a^2 + 2ab + b^2$.

28. $15(a^2b - ab^2)$, $21(a^3 - ab^2)$, and $35(ab^2 + b^3)$.

29. $x^2 + 5x + 6$, $x^2 - x - 12$, and $x^2 - 2x - 8$.

30. $x^2 - 8x + 15$, $x^2 - 4x - 5$, and $x^2 - 2x - 3$.

31. $x^2 + 4xy - 21y^2$, $x^2 - 2xy - 3y^2$, and $x^2 - 6xy - 7y^2$

32. $x^2 - 3x - 28$, $x^2 + x - 12$, and $x^2 - 10x + 21$.

33. $x^2 + 2x - 35$, $x^2 + x - 42$, and $x^2 - 11x + 30$.

FRACTIONS.

108. 1. When anything is divided into two equal parts, what are the parts called? What, when it is divided into three equal parts? Into four equal parts? Into m equal parts? Into n equal parts?

2. What does $\frac{a}{3}$ represent? $\frac{c}{4}$? $\frac{x}{5}$? $\frac{n}{7}$? $\frac{1}{m}$? $\frac{1}{n}$?

3. Express $\frac{1}{5}$ of a; $\frac{1}{3}$ of b; $\frac{1}{8}$ of c; $\frac{2}{3}$ of a; $\frac{3}{5}$ of d.

109. One or more of the equal parts of anything is called a **Fraction.**

An operation in division may also be expressed as a fraction by writing the *dividend* for the *numerator* and the *divisor* for the *denominator.*

110. A quantity no part of which is in the form of a fraction is called an **Entire Quantity.**

Thus, a, $3x$, $4a + 3b$, etc., are entire quantities.

111. A quantity composed of an entire quantity and a fraction is called a **Mixed Quantity.**

Thus, $3x + \frac{2y}{5}$, $3x - 3y + \frac{a+b}{c}$ are mixed quantities.

112. The sign written before the dividing line is called the **Sign of the Fraction.**

This sign belongs to the fraction as a *whole* and not to either the numerator or the denominator.

Thus, in $-\frac{a+b}{2}$ the sign of the *fraction* is $-$, but the signs of the quantities a, b, and 2 are $+$. The sign before the dividing line simply shows whether the fraction is to be added or subtracted.

REDUCTION OF FRACTIONS.

113. To reduce fractions to higher or lower terms.

1. How many fourths are there in 1 half? In 3 halves? In 5 halves? In a halves? In b halves? In n halves?

2. How many sixths are there in 1 third? In 2 thirds? In 8 thirds? In x thirds? In y thirds? In $a+b$ thirds?

3. Since $\frac{a}{2}$ is equal to $\frac{2a}{4}$; and $\frac{x}{3}$ is equal to $\frac{2x}{6}$; what may be done to the terms of a fraction without changing the value of the fraction?

4. How many thirds are there in $\frac{4}{6}$? How many halves are there in $\frac{4}{8}$? How many fifths are there in $\frac{8}{10}$? How many twelfths are there in $\frac{6a}{24}$? In $\frac{9a}{36}$? In $\frac{16a}{48}$?

5. Change $\frac{3a}{4}$, $\frac{5a}{2}$, $\frac{6}{8a}$, $\frac{3a}{16}$ to fractions whose denominator is $16a$.

6. Reduce to equivalent fractions whose numerator is $3x$, $\frac{1}{ax}$, $\frac{3}{by}$, $\frac{3}{ab}$, $\frac{3xy}{4xy^3}$, $\frac{xyz}{4ayz}$.

7. What else, besides multiplying them by the same quantity, may be done to the terms of a fraction, without changing the value of the fraction?

114. A fraction is expressed in its **Lowest Terms** when its numerator and denominator have no *common divisor*.

115. Principle. *Multiplying or dividing both terms of a fraction by the same quantity does not change the value of the fraction.*

116. To express a fraction in higher terms.

1. Change $\frac{2a}{5bc}$ to a fraction whose denominator is $15b^2c^2d$.

PROCESS.

$$\frac{2a}{5bc}$$

$$15b^2c^2d \div 5bc = 3bcd$$

$$\frac{2a \times 3bcd}{5bc \times 3bcd} = \frac{6abcd}{15b^2c^2d}$$

EXPLANATION. Since the fraction is to be changed to an equivalent fraction expressed in higher terms, both terms of the fraction must be multiplied by the same quantity, so that the value of the fraction may not be changed (Prin.). In order to produce the required denominator, the given denominator must be multiplied by $3bcd$; consequently the numerator must also be multiplied by $3bcd$.

2. Change $\frac{3x}{5}$ to a fraction whose denominator is 30.

3. Change $\frac{5mn}{8}$ to a fraction whose denominator is 24.

4. Change $\frac{4a^2b}{7}$ to a fraction whose denominator is 28.

5. Change $\frac{8axy}{15b^2c}$ to a fraction whose denominator is $45b^2c^3d^2$.

6. Change $\frac{2a-5b}{11}$ to a fraction whose denominator is $33x$.

7. Change $\frac{4m-3n^2}{16}$ to a fraction whose denominator is 48.

8. Change $\frac{2xy}{3a+2}$ to a fraction whose numerator is $4x^2y$.

9. Change $\frac{6ab}{5x-3y}$ to a fraction whose numerator is $12abc$.

10. Change $\frac{4a-5b}{3}$ to a fraction whose denominator is $12xy$.

11. Change $\frac{7}{a+b}$ to a fraction whose denominator is a^2-b^2.

12. Change $\frac{m+n}{m-n}$ to a fraction whose denominator is m^2-n^2.

13. Change $\frac{6a-7}{15}$ to a fraction whose denominator is 75.

14. Change $\frac{1}{m+n}$ to a fraction whose numerator is $m+n$.

15. Change $\frac{2a}{x+2}$ to a fraction whose numerator is $2ax-4a$.

16. Change $\frac{x-y}{x+y}$ to a fraction whose numerator is x^2-y^2.

117. To express a fraction in its lowest terms.

1. Reduce $\frac{15x^2y^2}{20xy^2}$ to its lowest terms.

PROCESS.

$$\frac{15x^2y^2}{20xy^2}=\frac{3x}{4}$$

EXPLANATION. Since the fraction is to be changed to an equivalent fraction expressed in its lowest terms, the terms of the fraction may be divided by any quantity that will exactly divide each of them (Prin.). Dividing by the quantity, $5xy^2$, the expression is reduced to its lowest terms, since the terms have now no common factor. Or, the terms may be divided by their highest common divisor.

Reduce the following to their lowest terms:

2. $\frac{3abc}{9ab^2}$.	5. $\frac{13m^2n}{39m^2n^2}$.	8. $\frac{17m^3nx^4}{68mn^2x^5}$.
3. $\frac{4xy^2z}{12x^2yz^2}$.	6. $\frac{112abx^2}{252abxy}$.	9. $\frac{125a^5x^3z^6}{625a^7x^5z^4}$.
4. $\frac{10abx}{35a^2bcx^2}$.	7. $\frac{35x^3y^5z^2}{105x^2y^4z^5}$.	10. $\frac{2x}{4x^2-6ax}$.

11. $\dfrac{3ab}{a^2-ab^2}$.

12. $\dfrac{a+b}{a^2-b^2}$.

13. $\dfrac{m-n}{m^2-n^2}$.

4 14. $\dfrac{a-b}{a^2-2ab+b^2}$.

15. $\dfrac{x^2-y^2}{x^2+2xy+y^2}$.

16. $\dfrac{x+1}{x^2+2x+1}$.

17. $\dfrac{ax-a}{ax^2-a}$.

18. $\dfrac{a^2+a-6}{a^2-9a+14}$.

19. $\dfrac{m^2+8m+15}{m^2-2m-15}$.

20. $\dfrac{x^2-2x-24}{x^2-12x+36}$.

21. $\dfrac{x^2+x-20}{x^2+4x-5}$.

22. $\dfrac{m-n}{m^3-n^3}$.

23. $\dfrac{x^2-6x-40}{x^2-3x-70}$.

24. $\dfrac{x^2-y^2}{x^3-y^3}$.

25. $\dfrac{x^2-1}{x^3+1}$.

26. $\dfrac{3(x+y)}{27(x^2-y^2)}$.

27. $\dfrac{2x-3y}{8x^3-27y^3}$.

28. $\dfrac{x^2-1}{2xy+2y}$.

118. To reduce an entire or mixed quantity to a fraction.

1. How many thirds are there in 2? In 5? In 7? In a?

2. How many fifths are there in 4? In 7? In m?

3. How many fourths are there in $3\frac{1}{4}$? In $5\frac{3}{4}$? In $x+\dfrac{x}{4}$?

1. Reduce $x+\dfrac{a}{y}$ to a fractional form.

PROCESS.

$$x=\frac{xy}{y}$$

$$x+\frac{a}{y}=\frac{xy}{y}+\frac{a}{y}=\frac{xy+a}{y}$$

EXPLANATION. Since 1 is equal to $\dfrac{y}{y}$, x is equal to $\dfrac{xy}{y}$; consequently $x+\dfrac{a}{y}=\dfrac{xy}{y}+\dfrac{a}{y}$ or $\dfrac{xy+a}{y}$.

RULE. *Multiply the entire part by the denominator of the fraction; to this product add the numerator when the sign of the fraction is plus, and subtract it when it is minus, and write the result over the denominator.*

If the *sign of the fraction* is $-$, the signs of all the terms in the numerator must be changed when it is subtracted.

The student must note very carefully that the *sign of the fraction* affects the *whole numerator* and not simply the first term.

Reduce the following to fractional forms:

2. $5x + \frac{3y}{2}$.
3. $4x - \frac{2y}{5}$.
4. $8x + \frac{y}{4}$.
5. $2x - \frac{6y}{10}$.
6. $x + \frac{2x-1}{7}$.
7. $2x - \frac{x+4}{3}$.
8. $7x + \frac{5y-2x}{6}$.
9. $8a - \frac{4z+x}{5}$.
10. $3m + \frac{2m-n}{3}$.
11. $x + \frac{a-b}{c}$.
12. $2x - \frac{y+z}{9}$.
13. $5b + \frac{3c-4b}{2}$.
14. $x - \frac{5y-2x}{7}$.
15. $3x + \frac{6a-x}{ax}$.
16. $a - \frac{2ac-c^2}{a}$.
17. $1 + \frac{x}{1+x}$.
18. $3 + \frac{3}{x^2-1}$.
19. $3a + \frac{ab-a}{b}$.
20. $a + x + \frac{a^2+x^2}{a-x}$.
21. $a + c + \frac{2ac-c^2}{a-c}$.
22. $2x - 3 - \frac{x^2+2}{x+2}$.
23. $m - 2n + \frac{14}{m+2n}$.
24. $m + n - x + \frac{x^2-5}{m+n+x}$.
25. $a - x + \frac{a^2+x^2-5}{a+x}$.
26. $a(m+n) + \frac{an^2}{m-n}$.

119. To reduce a fraction to an entire or mixed quantity.

1. How many units are there in $\frac{9}{3}$? In $\frac{5}{2}$? In $\frac{28}{5}$?

2. How many units are there in $\frac{3a+3b}{3}$? In $\frac{5x-10}{5}$? In $\frac{4m+n}{2}$?

1. Reduce $\frac{ax+b}{x}$ to a mixed quantity.

PROCESS.

$$\frac{ax+b}{x}=a+\frac{b}{x}$$

EXPLANATION. Since a fraction may be regarded as an expression of unexecuted division, by performing the division indicated, the fraction is changed into the form of a mixed quantity.

Reduce the following to entire or mixed quantities:

2. $\frac{23a}{9}$.
3. $\frac{26ab}{11}$.
4. $\frac{45x^2y^3z}{15xy}$.
5. $\frac{36ac+4c}{9}$.
6. $\frac{a^2+c^2}{a}$.
7. $\frac{12x^3-5y}{6x}$.
8. $\frac{2a^2x-ax^3}{a}$.
9. $\frac{a^2-x^2}{a-x}$.
10. $\frac{x^3+1}{x-1}$.
11. $\frac{2a^2-4b^2}{a+b}$.
12. $\frac{5ay+ax+x}{a}$.
13. $\frac{2ab+b^2}{a+b}$.
14. $\frac{2x^2+7}{x-4}$.
15. $\frac{x^3+y^3}{x-y}$.
16. $\frac{2ab+ab^2-a^2}{ab}$.
17. $\frac{x^3+2x^2-2x+1}{x^2-x-1}$.

18. $\frac{2a^2-2b^2}{a-b}$.

19. $\frac{x^2-2x+1}{x-1}$.

20. $\frac{2x^2+5}{x-3}$.

21. $\frac{a^2+b^2}{a-b}$.

22. $\frac{x^2-4x+6}{x-2}$.

23. $\frac{2x^2-6x+4}{2x-3}$.

24. $\frac{5x^2-7x+5}{5x-1}$.

25. $\frac{x^4-x^3-x+1}{x^2-1}$

120. To reduce dissimilar to similar fractions.

1. Into what fractions having the same fractional unit may $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ be changed?

2. Into what fractions having the same fractional unit may $\frac{1}{3m}$ and $\frac{1}{2m}$ be changed?

3. Express $\frac{1}{3m}$, $\frac{1}{2m}$, and $\frac{1}{6m}$ in equivalent fractions having their lowest common denominator.

121. Fractions which have the same fractional unit are called **Similar Fractions.**

122. Fractions which have not the same fractional unit are called **Dissimilar Fractions.** Similar fractions have, therefore, a common denominator.

123. When similar fractions are expressed in their lowest terms, they have their **Lowest Common Denominator.**

124. PRINCIPLES. 1. *A common denominator of two or more fractions is a common multiple of their denominators.*

2. *The lowest common denominator of two or more fractions is the lowest common multiple of their denominators.*

1. Reduce $\frac{m}{2x}$ and $\frac{3}{a^2x}$ to similar fractions having their lowest common denominator.

PROCESS.

$$\frac{m}{2x} = \frac{m \times a^2}{2x \times a^2} = \frac{a^2m}{2a^2x}$$

$$\frac{3}{a^2x} = \frac{3 \times 2}{a^2x \times 2} = \frac{6}{2a^2x}$$

EXPLANATION. Since the lowest common denominator of several fractions is the lowest common multiple of their denominators (Prin. 2), the lowest common multiple of the denominators $2x$ and a^2x must be found, which is $2a^2x$. The fractions are then reduced to fractions having the denominator $2a^2x$, by multiplying the numerator and denominator of each fraction by the quotient of $2a^2x$ divided by its denominator. $2a^2x \div 2x = a^2$, the multiplier of the terms of the first fraction. $2a^2x \div a^2x = 2$, the multiplier of the terms of the second fraction.

RULE. *Find the lowest common multiple of the denominators of the fractions for the lowest common denominator.*

Divide this denominator by the denominator of each fraction, and multiply the terms of the fraction by the quotient.

Reduce to similar fractions, having their lowest common denominator:

2. $\frac{a}{b}$ and $\frac{b}{c}$.

3. $\frac{m}{n}$ and $\frac{ab}{x}$.

4. $\frac{3xy}{c}$ and $\frac{2a}{3by}$.

5. $\frac{ab}{4}$ and $\frac{cd}{3y}$.

6. $\frac{2b}{x^2z}$ and $\frac{3cd}{xz^2}$.

7. $\frac{ax}{y}$, $\frac{bc}{ax}$, $\frac{ab}{ay}$.

8. a, $\frac{bc}{xy}$, $\frac{ac}{xz}$, $\frac{ab}{yz}$.

9. $\frac{2}{xy}$, $\frac{2}{ax}$, $\frac{2}{ayz}$.

10. $\dfrac{m}{a^2b}, \dfrac{n}{bc}, \dfrac{mn}{b^2c^2}.$

11. $\dfrac{10a}{xy}, \dfrac{8b}{x^2}, \dfrac{9}{xy^2z}.$

12. $\dfrac{2x}{13}, \dfrac{2x}{3}, \dfrac{xy}{26}, \dfrac{4y}{39}.$

13. $\dfrac{a+b}{3x}, \dfrac{a-b}{2y}, \dfrac{ab}{6}.$

14. $\dfrac{x+y}{4}, \dfrac{x-y}{2c}, \dfrac{x^2+y^2}{2a}.$

15. $\dfrac{5}{x+y}, \dfrac{7}{x-y}, \dfrac{x+y}{x^2-y^2}.$

16. $\dfrac{3}{a+b}, \dfrac{4}{a-b}.$

17. $\dfrac{3}{x+2}, \dfrac{5}{x-2}, \dfrac{9}{x}.$

18. $\dfrac{a+b}{a^3-b^3}, \dfrac{1}{a-b}, \dfrac{1}{a^2+ab+b^2}.$

19. $\dfrac{1}{x+1}, \dfrac{3}{4x+4}, \dfrac{x}{x^2-1}.$

20. $\dfrac{2}{a^2-b^2}, \dfrac{1}{a-b}, \dfrac{3}{a^2+b^2}.$

21. $\dfrac{x+2}{x^2-3x+2}, \dfrac{x-2}{x^2+x-2}.$

22. $\dfrac{x}{x^2+2x-3}, \dfrac{a}{x^2-2x-15}.$

23. $\dfrac{x+1}{x(x-2)}, \dfrac{x-1}{4x-8}, \dfrac{2}{4x}.$

24. $\dfrac{5}{1+2x}, \dfrac{3x}{1-2x}, \dfrac{4-13x}{1-4x^2}.$

25. $\dfrac{1}{a+b}, \dfrac{a}{a^2-b^2}, \dfrac{3a}{a^4-b^4}.$

26. $\dfrac{x-y}{x^2-xy+y^2}, \dfrac{x+y}{x^3+y^3}, \dfrac{x^2}{xy+y^2}.$

CLEARING EQUATIONS OF FRACTIONS.

125. 1. Five is one half of what number?

2. Eight is one third of what number? One fifth of what number?

3. If $\frac{1}{2}x$ equals 5, what is the value of x?

4. If $\frac{1}{5}x = 6$, what is the value of x?

5. What effect has it upon the equality of the members of an equation to multiply both by the same quantity?

6. If the members of the equation $\frac{1}{5}x = 6$ are multiplied by 5, what is the resulting equation?

7. Multiply the following equations by such quantities as will change them into equations without fractions:

$$\tfrac{1}{3}x = 8, \text{ or } \frac{x}{3} = 8; \quad \tfrac{1}{6}x = 7, \text{ or } \frac{x}{6} = 7;$$

$$\frac{x}{8} = 4; \quad \frac{x}{10} = 9; \quad \frac{x}{2} + \frac{x}{4} = 6; \quad \frac{x}{3} + \frac{x}{6} = 5;$$

$$\frac{x}{4} + \frac{x}{8} = 10; \quad \frac{x}{5} + \frac{x}{10} = 20; \quad \frac{x}{3} + \frac{x}{5} = 8.$$

8. How may an equation containing fractions be changed into an equation without fractions?

126. Changing an equation containing fractions into another equation without fractions is called **Clearing an Equation of Fractions.**

127. PRINCIPLE. *An equation may be cleared of fractions by multiplying both members by some multiple of the denominators of the fractions.*

1. Find the value of x in the equation $x+\frac{x}{3}=8$.

PROCESS.

$$x+\frac{x}{3}=8$$

Clearing of fractions, $3x+x=24$

Uniting terms, $4x=24$

Therefore, $x=6$

EXPLANATION. Since the equation contains a fraction, it may be cleared of fractions by multiplying each member by the denominator of the fraction (Prin.). The denominator is 3; therefore each member is multiplied by 3; thus, 3 times x is $3x$, 3 times $\frac{x}{3}$, or $\frac{1}{3}x$, is $\frac{3}{3}x$, or x, and 3 times 8 is 24. The resulting equation is $3x+x=24$; therefore $x=6$.

2. Find the value of x in the equation

$$x+\frac{x}{4}+\frac{x}{2}+\frac{x}{5}=\frac{39}{2}.$$

PROCESS.

$$x+\frac{x}{4}+\frac{x}{2}+\frac{x}{5}=\frac{39}{2}$$

Clearing of fractions, $20x+5x+10x+4x=390$

Therefore, $39x=390$

and $x=10$

EXPLANATION. Since the equation may be cleared of fractions by multiplying by some multiple of the denominators (Prin.), this equation may be cleared of fractions by multiplying both members by 4, 2, 5, and 2 successively, or by their product, or by any multiple of 4, 2, 5, and 2.

Since the multiplier will be the smallest when we multiply by the lowest common multiple of the denominators, we multiply both members by 20, the lowest common multiple of 4, 2, 5, and 2. Uniting terms and dividing by the coefficient of x, the result is $x=10$.

RULE. *Multiply both members of the equation by the least, or lowest, common multiple of the denominators.*

1. An equation may also be cleared of fractions by multiplying each member by the product of all the denominators.

2. Multiplying a fraction by its denominator removes the denominator.

3. If a fraction has the *minus* sign before it, *the signs of all the terms of the numerator must be changed when the denominator is removed.*

3. Find the value of x in the equation

$$\frac{x-3}{4}-\frac{x-1}{9}=\frac{x-5}{6}.$$

SOLUTION. $\qquad \frac{x-3}{4}-\frac{x-1}{9}=\frac{x-5}{6}$

Clearing of fractions, $\qquad 9x-27-4x+4=6x-30$

Transposing, $\qquad 9x-4x-6x=27-4-30$

Uniting terms, $\qquad -x=-7$

Dividing by -1, $\qquad x=7$

Note 3 under the rule is a very important one. Observe its application in the above solution.

Find the value of x and verify the result in the following:

4. $x+\frac{x}{4}=10.$

5. $x+\frac{x}{3}=20.$

6. $2x+\frac{x}{4}=9.$

7. $2x+\frac{x}{8}=34.$

8. $4x+\frac{x}{4}=51.$

9. $\frac{x}{3}+3x=10.$

10. $x+\frac{x}{2}+\frac{x}{3}=11.$

11. $\frac{x}{3}+\frac{x}{4}+\frac{x}{6}=9.$

12. $2x+\frac{x}{2}+\frac{x}{7}=37.$

13. $x+\frac{x}{3}+\frac{x}{5}=\frac{69}{5}.$

14. $x+\frac{x}{5}+\frac{x}{10}+\frac{x}{15}=41.$

15. $\frac{x}{3}+\frac{x}{6}+\frac{x}{8}+\frac{x}{12}=17.$

16. $3x+\frac{x}{9}+\frac{x}{12}=\frac{115}{12}.$

17. $2x+\frac{2x}{4}+\frac{3x}{15}=27.$

18. $3x-\frac{x}{6}+\frac{x}{12}=70.$

19. $x+\frac{x}{7}-\frac{x}{5}=33.$

20. $x+\frac{3x}{5}+\frac{2x}{6}-\frac{x}{2}=\frac{43}{2}.$

21. $\frac{2x}{3}-\frac{x}{5}+\frac{x}{4}+\frac{x}{12}=8.$

22. $2x+\frac{x}{8}-\frac{2x}{3}+\frac{x}{6}=\frac{13}{2}.$

23. $x+\frac{x}{2}-\frac{x}{4}+\frac{x}{11}=59.$

24. $\frac{x}{10}-\frac{2x}{5}+\frac{3x}{2}-\frac{x}{4}=\frac{19}{4}.$

25. $\frac{x}{3}+\frac{x+2}{8}=x-3.$

26. $\frac{x}{3}-\frac{x-1}{11}=x-9.$

27. $\frac{9x}{7}-2x=\frac{x+3}{5}-21.$

28. $\frac{3x+4}{3}=\frac{40}{3}-2x.$

29. $2-\frac{x+8}{4}=2x-\frac{x+6}{3}.$

30. $\frac{x+3}{2}=\frac{2x+3}{10}+\frac{7x-5}{5}.$

31. $\frac{7x+16}{21}=\frac{x+8}{21}+\frac{x}{3}.$

32. $\frac{x+3}{4}-\frac{x-8}{5}=\frac{x-5}{2}-1.$

33. $\frac{6x-14}{8}+\frac{2x-1}{3}=2x-5.$

34. $\frac{23x+13}{4}-\frac{x-1}{2}=6.$

35. $\frac{4x+2}{11}=\frac{3x-5}{13}+1.$

36. $\frac{x+1}{3}+\frac{x+3}{4}=\frac{x+4}{5}+16.$

37. $\frac{x-7}{10}+\frac{x-7}{5}=\frac{x+1}{6}.$

38. $\frac{x+3}{2}-\frac{x-1}{3}=\frac{x+42}{7}-\frac{x+5}{3}.$

39. $\frac{x}{2}+x=\frac{x}{3}+\frac{6x+1}{6}.$

40. $\frac{x}{3}-\frac{2x-4}{7}=x-\frac{x+7}{2}.$

41. $\frac{1-2x}{3}=\frac{4-5x}{4}-\frac{13}{42}.$

42. $\frac{x}{5}+\frac{x-2}{3}=x-\frac{5x-1}{6}.$

43. $\frac{x}{8}+\frac{x}{5}-\frac{2x}{5}=\frac{x-52}{4}.$

44. $\frac{x-4}{4}-\frac{x-1}{3}=\frac{x-26}{5}.$

45. $\frac{x-1}{2}+\frac{x-3}{4}-\frac{x-2}{3}=\frac{2}{3}.$

PROBLEMS.

128. 1. A certain number diminished by $\frac{1}{5}$ and also by $\frac{1}{6}$ of itself leaves a remainder of 19. What is the number?

SOLUTION. Let $x =$ the number.

Then $\frac{x}{5}$ and $\frac{x}{6}$ = the parts of the number.

And $$x-\frac{x}{5}-\frac{x}{6}=19$$

Clearing of fractions, $30x-6x-5x=570$

Uniting, $19x=570$

$$x=30$$

2. What number is that the sum of whose third and fourth parts equals its sixth part plus 5?

3. A man left $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his children, and the remainder, which was \$1200 to a public library. What was the value of his property?

4. Out of a cask of wine $\frac{1}{5}$ part leaked away; afterward 10 gallons were drawn out. The cask was then $\frac{2}{3}$ full. How many gallons did it hold?

5. A man leased some property for 40 years. $\frac{1}{2}$ of the time the lease has run is equal to $\frac{1}{3}$ of the time it has to run. How many years has it run?

6. The sum of two numbers is 35, and $\frac{1}{3}$ of the less is equal to $\frac{1}{4}$ of the greater. What are the numbers?

7. A man paid $\frac{2}{3}$ as much for a wagon as for a horse, and the price of the horse plus $\frac{1}{2}$ of the price of the wagon was 100 dollars. What was the price of each?

8. The sum of two numbers is 76, and $\frac{2}{5}$ of the less is equal to $\frac{1}{7}$ of the greater. What are the numbers?

9. There is a certain number the sum of whose fifth and seventh parts exceeds the difference of its seventh and fourth parts by 99? What is the number?

10. Divide the number 50 into two such parts that $\frac{1}{2}$ of one plus $\frac{3}{4}$ of the other shall equal 35.

11. A son's age is $\frac{2}{5}$ of his father's; but 15 years ago the son's age was $\frac{1}{7}$ of the father's. What are the ages of each?

12. A man paid \$ 3 a head for some sheep. After 20 of them had died, he sold $\frac{1}{4}$ of the remainder at cost for \$ 60. How many sheep did he buy?

13. A man wished to distribute some money among a certain number of children. He found that, if he gave to each of them 8 cents, he would have 10 cents left, but, if he gave to each 10 cents, he lacked 10 cents of having enough. How many children were there, and how much money had the man?

14. A man and his oldest son can earn \$ 30 a week; the man and his youngest son can earn $\frac{5}{6}$ as much; and the two sons can earn \$ 19 a week. How much can each alone earn per week?

15. A and B start in business with the same capital. A gains \$ 1775, and B loses \$ 225. B then has $\frac{2}{3}$ as much money as A. What was their original capital?

16. Divide \$ 440 between A, B, and C so that B shall have \$ 5 more than A, and C shall have $\frac{3}{5}$ as much as A and B.

ADDITION AND SUBTRACTION OF FRACTIONS.

129. 1. Find the value of $\frac{1}{2}+\frac{1}{6}$; of $\frac{1}{2}-\frac{1}{4}$; of $\frac{2a}{3}+\frac{a}{6}$.

2. What kind of fractions can be added or subtracted without changing their form?

3. What must be done to dissimilar fractions before they can be added or subtracted? How are dissimilar fractions made similar?

130. Principles. 1. *Only similar fractions can be added or subtracted.*

2. *Dissimilar fractions must be reduced to similar fractions before they can be united by addition or subtraction into one term.*

1. What is the sum of $\frac{a}{2}$, $\frac{2a}{5}$, and $\frac{3c}{4}$?

Solution. $\frac{a}{2}+\frac{2a}{5}+\frac{3c}{4}=\frac{10a}{20}+\frac{8a}{20}+\frac{15c}{20}=\frac{18a+15c}{20}$.

2. Subtract $\frac{2a}{3x}$ from $\frac{7b}{8y}$.

Solution. $\frac{7b}{8y}-\frac{2a}{3x}=\frac{21bx}{24xy}-\frac{16ay}{24xy}=\frac{21bx-16ay}{24xy}$.

3. Simplify $\frac{x+2}{x-2}+\frac{x-2}{x+2}-\frac{x^2+4}{x^2-4}$.

Solution. The lowest common denominator is x^2-4,

$$\frac{x+2}{x-2}+\frac{x-2}{x+2}-\frac{x^2+4}{x^2-4}$$

$$=\frac{x^2+4x+4}{x^2-4}+\frac{x^2-4x+4}{x^2-4}-\frac{x^2+4}{x^2-4}=\frac{x^2+4}{x^2-4}.$$

Add:

4. $\frac{2}{ab}$ and $\frac{3}{ab}$.

5. $\frac{2ab}{3xy}$ and $\frac{5ad}{3xy}$.

6. $\frac{x}{y}$ and $\frac{m}{n}$.

7. $\frac{x}{3a}$ and $\frac{3y}{6ax}$.

8. $\frac{a}{a+x}$ and $\frac{a}{a-x}$.

9. $\frac{a+b}{4}$ and $\frac{a^2}{a-b}$.

10. $\frac{2a+b}{3c}$ and $\frac{5a-4b}{12c}$.

11. $\frac{2x^2}{x^2-y^2}$ and $\frac{2x}{x+y}$.

12. $\frac{a+4}{a+3}$ and $\frac{a+2}{a+5}$.

13. $\frac{2x-3y}{2x-y}$ and $\frac{2x-y}{x-y}$.

14. $\frac{5}{m+n}$ and $\frac{7}{m-n}$.

15. $\frac{x+1}{x-1}$ and $\frac{x-1}{x+1}$.

16. $\frac{y^2-2xy-x^2}{x^2-xy}$ and $\frac{x}{x-y}$.

17. $\frac{-3xy}{x^3-y^3}$ and $\frac{3}{x-y}$.

18. $\frac{a-7}{a+2}$ and $\frac{a+4}{a-5}$.

19. $\frac{a-b}{a+b}$ and $\frac{a+b}{a-b}$.

20. $\frac{5}{2x-1}$ and $\frac{2x-7}{4x^2-1}$.

21. $\frac{1}{x+y}$ and $\frac{2y}{x^2-y^2}$.

22. $\frac{a}{x(a-x)}$ and $\frac{x}{a(a-x)}$.

23. $\frac{3a}{a+x}$ and $\frac{2ax}{a^2-x^2}$.

Remember to *change the signs of all the terms in the numerator* when you subtract a fraction.

Subtract:

24. $\frac{3x}{9}$ from $\frac{5x}{9}$.

25. $\frac{b}{5}$ from $\frac{2a}{5}$.

26. $\frac{mn}{ax}$ from $\frac{bc}{y}$.

27. $\frac{2b}{xy}$ from $\frac{cd}{yz}$.

28. $\frac{1}{x-1}$ from $\frac{1}{x-2}$.

29. $\frac{2x-3}{x^2-4}$ from $\frac{5}{x-2}$.

30. $\frac{a-b}{a^2-b^2}$ from $\frac{a+b}{a^2-2ab+b^2}$.

31. $\frac{3}{x+2}$ from $\frac{x}{x+3}$.

32. $\frac{5}{x^2-2x}$ from $\frac{6}{2x}$.

33. $\frac{1}{(x-a)^2}$ from $\frac{1}{(x+a)^2}$.

34. Find the sum of one half of c dollars and one third of c plus d dollars.

35. A man having b dollars paid out one half of his money, and then one fifth of the remainder. How much did he pay out? How much had he left?

36. A carpet cost a dollars, a table b dollars, and a desk one half as much as the carpet and the table. What was the cost of all?

Simplify:

37. $\frac{2xy}{15}+\frac{4xy}{5}-\frac{3xy}{7}$.

38. $\frac{2}{3}x-\frac{1}{6}x+\frac{1}{4}x$.

39. $\frac{5}{11}ab-\frac{1}{2}ab+\frac{1}{3}ab$.

40. $\frac{m}{3}+\frac{2m}{4}-\frac{5n}{6}$.

41. $\frac{3x}{5}+\frac{x}{4}+\frac{3x}{10}$.

42. $\frac{2a}{7}-\frac{4b}{9}+\frac{a}{3}$.

43. $\frac{5ax}{6}-\frac{2ax}{9}+\frac{5ax}{12}$.

44. $\frac{a-b}{3}+\frac{a+b}{4}-\frac{a}{2}+\frac{b}{5}$.

45. $\frac{x}{y}+\frac{y}{x+y}+\frac{x^2}{x^2+xy}$.

46. $\frac{xy}{x+y}-\frac{xy}{x-y}+\frac{1}{x-y}$.

47. $\frac{1}{1+a}+\frac{1}{1-a}+\frac{2a}{1-a^2}$.

48. $\frac{a-b}{ab}+\frac{b-c}{bc}-\frac{c-a}{ac}$.

49. $\frac{2x}{x^2-1}-\frac{3}{x+1}+\frac{5}{x-1}$.

50. $\dfrac{2x-5}{x^2-x-2}-\dfrac{3}{x-2}+\dfrac{7}{x+1}.$

51. $\dfrac{1}{x+1}+\dfrac{2}{x+2}+\dfrac{3}{x-3}.$

52. $\dfrac{1}{x^2-x-2}-\dfrac{1}{x^2+x-2}.$

53. $x-\dfrac{x^2}{x+1}+\dfrac{x}{x-1}.$

54. $\dfrac{1}{x-a}+\dfrac{1}{x+a}-\dfrac{2}{x}.$

55. $\dfrac{a}{a-x}+\dfrac{3a}{a+x}-\dfrac{2ax}{a^2+x^2}.$

56. A man walked a miles the first day; the second day he walked half as far as the first day; and the third day half as far as the second. How far did he walk in the 3 days?

57. What is the cost of a coat, hat, and pair of shoes, if the coat costs a dollars, the hat c dollars, and the shoes $\frac{2}{5}$ as much as the hat and coat?

58. A train went e miles the first hour, f miles the second, and then went back $\frac{1}{8}$ of the whole distance. How far was the train then from the starting point?

MULTIPLICATION OF FRACTIONS.

131. 1. How much is three times $\frac{2}{7}$? Four times $\frac{3}{5}$? Five times $\dfrac{2a}{9}$?

2. Express $2\times\frac{3}{4}$ in its lowest terms; $5\times\frac{7}{15}$; $8\times\frac{5}{24}$; $3\times\dfrac{2a}{9}$.

3. How may the products in 1 and 2 be obtained from the given fractions?

4. In what two ways, then, may a fraction be multiplied by an integer?

5. How much is $\frac{1}{3}$ of $\frac{3}{5}$ or $\frac{3}{5}\div 3$? $\frac{1}{2}$ of $\dfrac{4a}{5}$ or $\dfrac{4a}{5}\div 2$?

6. In what two ways, then, may a fraction be divided by an integer?

132. PRINCIPLES. 1. *Multiplying the numerator or dividing the denominator of a fraction by any quantity, multiplies the fraction by that quantity.*

2. *Dividing the numerator or multiplying the denominator of a fraction by any quantity, divides the fraction by that quantity.*

1. What is the product of $\frac{x}{y}$ multiplied by $\frac{c}{3}$?

PROCESS.

$$\frac{x}{y} \times \frac{c}{3} = \frac{cx}{3y}$$

EXPLANATION. To multiply $\frac{x}{y}$ by $\frac{c}{3}$ is to find c times $\frac{1}{3}$ part of $\frac{x}{y}$. $\frac{1}{3}$ part of $\frac{x}{y}$ is $\frac{x}{3y}$ (Prin. 2), and c times $\frac{x}{3y}$ is $\frac{cx}{3y}$ (Prin. 1). That is, *the product of the numerators is the numerator of the product, and the product of the denominators is the denominator of the product.*

1. Reduce all entire and mixed quantities to the fractional form before multiplying.

2. Entire quantities may be expressed in the form of a fraction by writing 1 as a denominator. Thus, a may be written $\frac{a}{1}$.

3. When possible, factor the terms of the fractions and *cancel* equal factors from numerator and denominator.

2. Find the product of $\frac{x-3}{x^2-4} \times \frac{x^2-1}{4x} \times \frac{x+2}{x^2-2x-3}$.

SOLUTION. $\frac{x-3}{x^2-4} \times \frac{x^2-1}{4x} \times \frac{x+2}{x^2-2x-3}$

$$= \frac{x-3}{(x+2)(x-2)} \times \frac{(x+1)(x-1)}{4x} \times \frac{x+2}{(x-3)(x+1)}$$

Canceling equal factors from dividend and divisor, the result is $\frac{x-1}{4x(x-2)}$.

Multiply:

3. $\frac{m^2n}{abcx}$ by abn.

4. $\frac{5c^2d}{27ax^2}$ by $6a$.

5. $\frac{3a}{4b}$ by $\frac{d}{a}$.

6. $\frac{2m}{5ax}$ by $\frac{ab}{xyz}$.

7. $\frac{7am}{9bn}$ by $\frac{3mn}{2ax}$.

8. $\frac{x+y}{abc}$ by $\frac{bc}{ax}$.

9. $\frac{m-n}{18}$ by $\frac{6}{m+n}$.

10. $\frac{11m^2n^2}{x^2-y^2}$ by $x+y$.

11. $\frac{2a}{x+y}$ by $\frac{5ab}{x-y}$.

12. $\frac{3ab}{x^2-4}$ by $\frac{x-2}{a}$.

13. $\frac{a-x}{x^2}$ by $\frac{a^2}{a^2-x^2}$.

14. $\frac{x+4}{5x-15}$ by $\frac{x-3}{x^2+5x+4}$.

15. $\frac{a^2-ab}{a^2-2ab+b^2}$ by $\frac{a^2-4ab+3b^2}{a^2-b^2}$.

16. $\frac{x^2-x-2}{x^2+4x+3}$ by $\frac{x^2-3x-10}{x^2-4}$.

17. $\frac{3m-12}{m^2+3m+2}$ by $\frac{m+2}{m^2-5m+4}$.

18. $\frac{x+3}{5x-15}$ by $\frac{x^2-9}{x^2-4x-21}$.

19. $\frac{20}{x^2-64}$ by $\frac{x^2+3x-40}{5x-25}$.

20. $\frac{a^4-b^4}{a^2b^2}$ by $\frac{ab^3}{a^2+b^2}$.

21. $\frac{x^2+2x-63}{x^2-4x-21}$ by $\frac{x^2+9x+18}{6x+54}$.

Simplify the following:

22. $\frac{a}{bc} \times \frac{c^2}{ab} \times \frac{b^2}{ac}$.

23. $\frac{x^2 - y^2}{x} \times \frac{x}{x+y} \times \frac{a}{x-y}$.

24. $\left(2 - \frac{x^2}{8}\right)\left(\frac{4}{x+4}\right)$.

25. $\left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{x^2y}{x^2 - y^2}\right)$.

26. $\left(\frac{x+5}{x^2 + 10x + 25}\right)\left(\frac{x^2 - 25}{x-2}\right)\left(\frac{x^2 - 4}{x-5}\right)$.

27. $\left(\frac{x}{2} + \frac{x}{3} - \frac{2y}{4}\right)\left(\frac{3xy}{5x^2 - 3xy}\right)$.

28. $\frac{4x^2y}{9xz} \times \frac{2ax}{x^2 - y^2} \times \frac{3(x+y)}{8xy^2} \times \frac{x-y}{xyz}$.

29. $\left(\frac{1}{m^2 + m + 1}\right)\left(\frac{1}{m^2} + \frac{1}{m} + 1\right)$.

30. $\frac{a+1}{a+3} \times \frac{a-2}{a-5} \times \frac{a^2 - 2a - 15}{a^2 - a - 2}$.

31. If a cords of wood cost b dollars, how much will c cords cost at the same rate?

32. If b pounds of sugar are worth c pounds of butter, how many pounds of butter are 6 pounds of sugar worth?

33. A father works a weeks at n dollars per week, and his son works 4 weeks at $\frac{1}{3}$ of the wages that the father receives. How much do both receive?

34. What is the value of d acres of land at $\frac{2}{3}$ of $(d - e)$ dollars per acre?

35. A walked c miles in b hours. How far did he walk in 1 hour? In 5 hours? In a hours? In $c + d$ hours?

DIVISION OF FRACTIONS.

133. 1. How many times is $\frac{1}{5}$ contained in 1? $\frac{1}{6}$ in 1? $\frac{1}{12}$ in 1? $\frac{1}{15}$ in 1? $\frac{1}{a}$ in 1? $\frac{1}{3b}$ in 1? $\frac{1}{m+n}$ in 1?

2. How does the number of times a fraction, having 1 for its numerator, is contained in 1 compare with its denominator?

3. How many times is $\frac{3}{4}$ contained in 1? $\frac{5}{8}$ in 1? $\frac{5}{c}$ in 1? $\frac{2}{d}$ in 1?

1. What is the value of $\frac{b}{2c} \div \frac{a}{d}$?

PROCESS.

$$\frac{b}{2c} \div \frac{a}{d} = \frac{b}{2c} \times \frac{d}{a} = \frac{bd}{2ac}$$

EXPLANATION. The fraction $\frac{1}{d}$ is contained in 1 d times; and $\frac{a}{d}$ is contained in 1, $\frac{1}{a}$ part of d times, or $\frac{d}{a}$ times. And, since $\frac{a}{d}$ is contained in 1 $\frac{d}{a}$ times, it will be contained in $\frac{b}{2c}$, $\frac{b}{2c}$ times $\frac{d}{a}$ or $\frac{bd}{2ac}$ times. That is, *the quotient of one fraction divided by another is equal to the product of the dividend by the divisor inverted.*

1. Change entire and mixed quantities to the fractional form.
2. An entire quantity may be expressed as a fraction by writing 1 for its denominator.
3. When possible, factor the terms of the fractions and use cancellation.

2. Divide $\frac{x^2-1}{x^2+x-20}$ by $\frac{x+1}{x^2-25}$.

SOLUTION. $\frac{x^2-1}{x^2+x-20} \div \frac{x+1}{x^2-25} = \frac{(x+1)(x-1)}{(x+5)(x-4)} \times \frac{(x+5)(x-5)}{x+1}$

Canceling equal factors, the quotient is

$$\frac{(x-1)(x-5)}{x-4} \text{ or } \frac{x^2-6x+5}{x-4}$$

Divide:

3. $\frac{x}{y}$ by $\frac{m}{n}$.

4. $\frac{ab}{xy}$ by $\frac{b}{c}$.

5. $\frac{x^2y}{z}$ by $\frac{xy}{z^2}$.

6. $\frac{15\,ab}{6\,x^2}$ by $\frac{5\,x}{3\,b}$.

7. $\frac{4\,a^3x}{6\,dy^2}$ by $\frac{2\,a^2x^2}{8\,a^2y}$.

8. $\frac{5\,x^2y^3z}{6\,a^2b^2c}$ by $\frac{2\,ab^2}{15\,xy^2}$.

9. $\frac{1}{x^2-y^2}$ by $\frac{1}{x-y}$.

10. $\frac{a-b}{x}$ by a^2-b^2.

11. $\frac{a^2-4\,x^2}{a^2+4\,x^2}$ by $a-2\,x$.

12. $\frac{x^2y}{x^2-y^2}$ by $\frac{xy^2}{x+y}$.

13. $\frac{dc+dy}{2\,y}$ by $\frac{c+y}{xy^2}$.

14. $\frac{4}{x^2-x-2}$ by $\frac{2}{x+1}$.

15. $\frac{14}{x^3-y^3}$ by $\frac{7}{x-y}$.

16. $\frac{ax-bx}{25\,x^3}$ by $\frac{a-b}{5\,x^2}$.

17. $\frac{b^3+1}{ab}$ by $\frac{b^2-b+1}{axy}$.

18. $\frac{abx}{x^2-2\,x+1}$ by $\frac{bm}{x-1}$.

19. $\frac{5\,bc}{x^2-5\,x}$ by $\frac{3\,ac}{x^2-15\,x+50}$.

20. $\frac{1}{a^2-x^2}$ by $\frac{1}{(a+x)^2}$.

21. $\frac{2\,x}{x^2-7\,x}$ by $\frac{3\,x^2}{x^2-13\,x+42}$.

22. $\frac{x^2-4\,x+4}{x^2-2\,x-15}$ by $\frac{x^2-6\,x+8}{x^2+4\,x+3}$.

23. $\frac{a^2-10\,a+24}{a^2+5\,a-14}$ by $\left(\frac{a-6}{a-2}\times\frac{a-4}{a^2+6\,a-7}\right)$.

24. $\left(\frac{x}{2}+\frac{y}{3}\right)$ by $\frac{(3\,x+2\,y)\,(x-y)}{6\,(x+y)}$.

25. $4\,x^2-4\,y^2$ by $\frac{x+y}{x-y}$.

26. $\dfrac{x^2-y^2}{m^3+n^3}$ by $\dfrac{x-y}{m+n}$.

27. $\left(x-4+\dfrac{6}{x+1}\right)$ by $\left(x-\dfrac{6}{x+1}\right)$.

28. What is the cost of b tons of coal if 7 tons cost a dollars?

29. How many times a dollars are two times c dollars?

30. A man sells 4 horses at m dollars each, and after using n dollars of the money, divides the remainder equally among c children. What represents the share of each child?

31. A man exchanges d horses for f cows. At that rate what is the value of a horses?

32. A son's age is $\dfrac{1}{m}$ of his father's. The father's age is represented by $\frac{2}{3}$ of the difference between a and b. What represents the age of the son?

33. A man buys hay for b dollars a ton. When the market price of hay has risen \$3 a ton, how many tons will he sell for c dollars? How many tons will yield a profit of d dollars?

34. John has h marbles, which is 5 more than c times as many as his brother has. How many marbles has his brother?

35. A train has n hours in which to make a run of $3s$ miles. How many miles an hour must it travel?

36. A train has d hours in which to make a run of $3c$ miles. If it goes $\frac{2}{3}$ of the way in a hours, what must be its rate for the remainder of the journey?

37. Mary had 25 cents. She bought a oranges at b cents each, c apples at d cents each, and with the remainder of her money flags at f cents each. How many flags did she buy?

SIMPLE EQUATIONS.

134. An equation which contains, when reduced to its simplest form, only the first power of one unknown quantity in any term is called a **Simple Equation.**

Thus, $7x + 5 = 8x + 23$, and $x^2 + 5x + 5 = x^2 + 2x + 10$, which reduces to $3x = 5$, are simple equations.

135. Solve the following equations:

1. $4(x-1)=5(2x-8)$.
2. $x-6+5x=6+2x$.
3. $x(x-4)+60=(x-4)(x+30)$.
4. $(2x+6)(x+1)=(2x-2)(x+9)$.
5. $(x-1)(x-2)-(x-3)(x-4)=2$.
6. $(x+3)(x-2)-(x+1)(x-5)=9$.
7. $(x+4)(4-x)+(x+2)(x-3)=0$.
8. $(x+1)(1-x)+(x+5)(x-2)=0$.
9. $(x+3)(x-7)-(x+2)(x+4)=21$.
10. $(x+5)(x+4)-(x-1)(x+8)=36$.
11. $(x+6)(x+1)+(7+x)(3-x)=45$.
12. $(x+10)(x-11)-(x-1)(x+10)=20$.
13. $ax-bx=a^2-b^2$.
14. $bx+c^2=b^2+cx$.
15. $b^2+mx=m^2-bx$.
16. $x(1-3c)+9c^2=1$.
17. $x-1+4b=b(3b+x)$.
18. $a(a-4+x)=2(x-2)$.
19. $c(c-x+2)=5(7-x)$.
20. $a(x-a)=2n(x-2n)$.
21. $a(8+a)=4(x-4)+ax$.
22. $a(a-x)+5(1+x)=6a$.
23. $b^2-3a(3a-x)+bx=0$.

24. $3+\frac{x+2}{2}=\frac{2x}{5}+\frac{x}{2}.$

25. $\frac{2x}{3}+x=\frac{5x}{2}-10.$

26. $\frac{x+1}{2}+\frac{3x-1}{7}=x.$

27. $x+\frac{x+2}{2}=\frac{5x}{2}-\frac{3x}{4}.$

28. $\frac{x+4}{5}-2x=\frac{x}{3}-\frac{x+7}{6}.$

29. $\frac{x}{4}+\frac{3x}{2}-\frac{x+4}{5}=x+8.$

30. $1-\frac{2x}{3}=x-2-\frac{3x}{2}.$

31. $\frac{x}{2}+\frac{3x-10}{2}-\frac{x}{3}=0.$

32. $2x-\frac{x}{2}=\frac{x+10}{11}+\frac{3x-4}{2}.$

33. $\frac{x-4}{3}+\frac{x+1}{7}=x-\frac{5x-4}{6}.$

34. $\frac{x+5}{2}+x=\frac{x-5}{5}+\frac{7x+5}{4}.$

35. $\frac{x+1}{3}-\frac{2-x}{4}=\frac{1+x}{2}-\frac{x}{12}.$

36. $\frac{x-1}{4}+\frac{x+3}{3}=x+\frac{3-x}{2}.$

37. $\frac{x-25}{3}-\frac{x+25}{25}=20.$

38. $\frac{x-1}{2}+\frac{x-3}{4}-\frac{x-2}{3}=\frac{2}{3}.$

39. $\frac{2x}{5}-\frac{3x}{4}+\frac{x}{2}=\frac{x+5}{10}-\frac{x}{5}.$

40. $\frac{x+5}{3}-x=\frac{x}{2}-\frac{x+8}{6}+\frac{x}{4}.$

41. $\frac{x-3}{6}+\frac{x-6}{3}-\frac{x-5}{4}=0.$

42. $\frac{1-x}{3}+\frac{3x-5}{4}+\frac{x}{6}=\frac{1}{12}.$

43. $x-10=\frac{x+1}{4}-\frac{3x-1}{5}.$

44. $\frac{x+4}{2}+\frac{x+6}{4}+\frac{x+8}{6}=3.$

45. $\frac{x-3}{3}-\frac{x+2}{5}-1=\frac{2-3x}{10}.$

46. $\frac{2x}{5}+\frac{5x}{2}=\frac{7x+8}{10}+24.$

47. $\frac{x}{7}+\frac{2x+1}{3}=\frac{x+4}{6}+\frac{x-2}{2}.$

48. $\frac{2x-6}{5}+\frac{3x+1}{10}=\frac{1}{2}+\frac{3x-4}{5}.$

49. $\frac{3x-5}{2}-\frac{x+1}{4}=\frac{2x}{7}+\frac{5x-11}{6}.$

50. $\frac{2x+1}{9}+\frac{x-3}{2}=\frac{x-5}{6}+\frac{2x-2}{4}.$

51. $\frac{2x-8}{5}-\frac{x+4}{9}=\frac{x}{2}-5.$

52. $\frac{x}{a}-b=\frac{c}{d}-x.$

53. $\frac{ax-b}{c}+a=\frac{x+ac}{c}.$

54. $\frac{1}{x-1}-\frac{1}{x+1}=\frac{2x}{x^2-1}.$

55. $\frac{4}{x-2}+\frac{12}{x+2}=\frac{48}{x^2-4}.$

56. $\frac{x-12}{x-7}+\frac{x-4}{x-12}=2+\frac{7}{x-7}.$

57. $\frac{x}{3}-\frac{x^2-5x}{3x-7}=\frac{2}{3}.$

58. $\frac{x+7}{2}-\frac{x+5}{3}=5-\frac{5x+3}{4}.$

59. $\frac{10x+3}{3}-\frac{6x-7}{2}=10(x-1).$

60. $\frac{7x+16}{21}-\frac{x}{3}=\frac{x+8}{4x-11}.$

Suggestion. The equation may be expressed as follows:

$$\frac{7x}{21}+\frac{16}{21}-\frac{x}{3}=\frac{x+8}{4x-11}.$$

Simplifying,
$$\frac{16}{21}=\frac{x+8}{4x-11}.$$

61. $\frac{20x}{25}+\frac{36}{25}+\frac{5x+20}{9x-16}=\frac{4x}{5}+\frac{86}{25}.$

62. $1-\frac{x}{x+2}=\frac{4}{x+6}.$

63. $\frac{5x}{x+6}+\frac{10}{x-1}=5.$

64. $\frac{15}{x+3}-\frac{1}{3(x+3)}=\frac{44}{15}.$

65. $\frac{3x-2}{2x-5}-\frac{6x+13}{10}=\frac{21-3x}{5}.$

66. $\frac{7}{x-1}-\frac{6x-1}{x+1}=\frac{3(16-2x^2)}{x^2-1}.$

67. $\frac{10x+17}{18}-\frac{12x+2}{13x-16}=\frac{5x-4}{9}.$

See suggestion, Example 60.

68. $\frac{18x+19}{28}+\frac{11x+21}{6x+14}=\frac{9x+15}{14}.$

69. $\frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}.$

70. $\frac{2x-5}{16}+\frac{x+5}{4}=\frac{x+13}{8}+\frac{1}{16}.$

71. $\frac{4x+3}{9}=\frac{8x+19}{18}-\frac{7x-29}{5x-12}.$

72. $\frac{4}{x+2}+\frac{7}{x+3}=\frac{37}{x^2+5x+6}.$

73. $\frac{2-4x}{1-3x}+\frac{1}{6}=\frac{1-3x}{1-2x}.$

74. $\frac{4}{x-4}-\frac{10}{x-5}=\frac{2}{x^2-9x+20}.$

75. $\frac{a(x-a)}{b}+\frac{b(x-b)}{a}=x.$

76. $\frac{a}{bx}+\frac{b}{ax}=a^2+b^2.$

PROBLEMS.

136. 1. The sum of two numbers is 100, and the less number is 10 more than $\frac{1}{2}$ of the greater. What are the numbers?

2. A, B, and C build 216 rods of fence. A builds 7 rods a day, B builds 6 rods, and C builds $5\frac{1}{2}$ rods. If A and B work the same number of days and C works twice as many days as the others, how many days does each work?

3. A man bought a house for \$6000. At the time of the purchase he paid a certain amount in cash, and each year thereafter until the house was paid for he paid $\frac{1}{5}$ of his cash payment and \$400 besides. If in this way he paid the whole amount in 3 years, what was his cash payment?

4. A boy bought a certain number of apples at the rate of 3 for 2 cents. He sold them at the rate of 6 for 5 cents, and gained 12 cents. How many apples did he buy?

5. A lady bought two pieces of cloth; the longer piece lacked 4 yards of being three times the length of the shorter. She paid \$2 per yard for the longer piece, and \$$2\frac{1}{2}$ for the shorter, and the shorter piece cost just $\frac{1}{2}$ as much as the longer. How many yards were there in each piece?

6. The distance around a rectangular field is equal to 10 rods more than five times the breadth; and the length is $1\frac{2}{3}$ times the breadth. What are the length and the breadth of the field?

SUGGESTION. Let $x =$ the number of rods in the breadth of the field; then, $1\frac{2}{3}x =$ the number of rods in the length.

7. A man spends $\frac{1}{4}$ of his annual income for his board, $\frac{1}{8}$ for traveling expenses, $\frac{1}{10}$ for clothes, $\frac{1}{20}$ for other expenses, and saves \$380. What is his annual income?

8. Of a company of soldiers $\frac{3}{5}$ are on duty, $\frac{1}{2}$ of the remainder are absent on leave, $\frac{1}{10}$ of the whole are sick, and

the remaining 50 are off duty. How many soldiers are there in the company?

9. A farm of 450 acres was divided among A, B, and C so that A had $\frac{2}{3}$ as many acres as B, and C had $\frac{1}{2}$ as many acres as A and B together. How many acres had each?

10. A boy bought a certain number of apples at the rate of 2 for 1 cent. He sold half of the number for 1 cent apiece, and the other half at the rate of 2 for 1 cent. He gained 10 cents by the transaction. How many apples did he buy?

SUGGESTION. Let $2x$ equal the number of apples.

11. The length of a certain field is twice the breadth. If the length were $\frac{2}{3}$ as much, and the breadth $1\frac{1}{3}$ as much, the entire distance around the field would be 64 rods. What are the length and the breadth of the field?

12. Find a number whose half, third, and fourth added together equal the number plus 2.

13. A man in business gained \$100, and then lost $\frac{1}{2}$ of all that he had. He afterward gained \$150, when he found that his money was just equal to his original capital. What was the original capital?

14. Two numbers are to each other as 3 to 4. If 10 is subtracted from each, the smaller one will be $\frac{2}{3}$ of the larger. What are the numbers?

SUGGESTION. Let $3x$ and $4x$ represent the numbers.

15. A woman sold some eggs at 2 cents apiece; but when she came to deliver them, four of the eggs being broken, she received only 192 cents. How many eggs had she?

16. Two numbers are to each other as 5 to 7. One half of the first plus the second equals one half of the second plus 12. What are the numbers?

17. On Monday I sold 80 quarts of berries at a certain price per quart; on Wednesday 48 quarts at $\frac{3}{4}$ that price per quart; and on Friday 60 quarts at $\frac{7}{8}$ that price per quart. In all for berries that week I received \$6.74. What was the selling price of one quart on Monday?

18. A man gained in business as follows: The first year \$400 less than his original capital; the second year \$300 less than the original capital; and the third year \$200 less than the first year. The gains of the three years amounted to \$700 more than the capital. What was the original capital?

19. A mason, 5 carpenters, and 3 assistants receive together \$191 for working a certain number of days. The mason receives \$3 per day, the carpenters \$$2\frac{1}{2}$, and the assistants \$$1\frac{1}{5}$. How many days do they work?

20. A man spent \$300 more than $\frac{1}{2}$ of his earnings each year. In 5 years he had saved \$1000. How much did he earn each year?

21. A boy bought a certain number of apples at the rate of 4 for 5 cents, and sold them at the rate of 3 for 4 cents. He gained 60 cents. How many did he buy?

22. A boy spent $\frac{1}{2}$ of his money and 2 cents more. He then spent $\frac{1}{2}$ of what was left and 2 cents more, when he found that there remained 12 cents of his money. How much had he at first?

23. A man bought a horse and wagon, paying twice as much for the horse as for the wagon. He sold the horse at a gain of 50 per cent, and the wagon at a loss of 10 per cent. He received for both \$195. What did he pay for each?

24. A person engaged to reap a field of grain for \$$1\frac{1}{5}$ per acre; but leaving 6 acres not reaped, he received \$12. How many acres of grain were there?

SIMPLE SIMULTANEOUS EQUATIONS.

137. Elimination by Comparison.

1. $x + 3y = 5$ and $x - y = 1$ are simultaneous equations. Transpose the term containing y in each to the second member. What are the resulting equations?

2. Since the second members of these derived equations are each equal to x, how do they compare with each other? (Ax. 1.)

3. If these second members are formed into an equation, how many unknown quantities does it contain?

4. How, then, may an unknown quantity be eliminated from two simultaneous equations by *comparison?*

1. Given $\left\{\begin{array}{l} 2x - y = 3 \\ x + 3y = 19 \end{array}\right\}$ to find the values of x and y by comparison.

Solution.	$2x - y = 3$	(1)
	$x + 3y = 19$	(2)
Transposing in (1),	$2x = 3 + y$	(3)
Dividing by the coefficient of x,	$x = \dfrac{3+y}{2}$	(4)
Transposing in (2),	$x = 19 - 3y$	(5)
Equating (4) and (5) (Ax. 1),	$\dfrac{3+y}{2} = 19 - 3y$	(6)
Clearing of fractions,	$3 + y = 38 - 6y$	(7)
Transposing, etc.,	$7y = 35$	(8)
Dividing by the coefficient of y,	$y = 5$	(9)
Substituting the value of y in (5),	$x = 19 - 15$	(10)
	$x = 4$	(11)

RULE. *Find an expression for the value of the same unknown quantity in each equation. Make an equation of these values and solve it.*

Solve the following by elimination by comparison:

2. $\begin{cases} 4x + y = 10 \\ x + y = 7 \end{cases}$

3. $\begin{cases} 6x - y = 2 \\ x + 3y = 32 \end{cases}$

4. $\begin{cases} x + 2y = 11 \\ 6x - y = 40 \end{cases}$

5. $\begin{cases} x + 10y = 14 \\ 2x - y = 7 \end{cases}$

6. $\begin{cases} 2x - 2y = -6 \\ 3x - 2y = 6 \end{cases}$

7. $\begin{cases} x - y = 4 \\ 2x - y = 22 \end{cases}$

8. $\begin{cases} 3x - y = 14 \\ 2x + 2y = 28 \end{cases}$

9. $\begin{cases} 3x + y = 26 \\ x + 2y = 27 \end{cases}$

10. $\begin{cases} x + y = 9 \\ 2x + 3y = 20 \end{cases}$

11. $\begin{cases} 2x - y = 7 \\ 3x + 2y = 28 \end{cases}$

12. $\begin{cases} 3x - 2y = 2 \\ 2x - y = 5 \end{cases}$

13. $\begin{cases} 4x - 3y = 1 \\ 2x + y = 23 \end{cases}$

14. $\begin{cases} 2x + y = 27 \\ 3x - y = 3 \end{cases}$

15. $\begin{cases} 3x + 2y = 30 \\ 5x + y = 29 \end{cases}$

16. $\begin{cases} \dfrac{6}{x} + \dfrac{8}{y} = 1 \\ \dfrac{12}{x} + \dfrac{4}{y} = 1 \end{cases}$

17. $\begin{cases} ax + by = m \\ ax + cy = n \end{cases}$

18. $\begin{cases} \dfrac{x}{2} + \dfrac{y}{3} = 10 \\ \dfrac{x}{5} - \dfrac{y}{5} = -1 \end{cases}$

19. $\begin{cases} 2x + ay = b \\ ax + 2y = c \end{cases}$

20. $\begin{cases} \dfrac{x}{3} + \dfrac{y}{2} = 7 \\ \dfrac{x}{9} + \dfrac{y}{2} = 5 \end{cases}$

21. $\begin{cases} \dfrac{x}{5} + 5y = 51 \\ \dfrac{y}{5} + 5x = 27 \end{cases}$

138. Elimination by Substitution.

1. In the equation $x + y = 12$, how may the value of x be found if y equals 5?

2. $x + 2y = 8$ and $2x + 3y = 13$ are simultaneous equations. Find the value of x in the first equation by transposing $2y$ to the second member.

3. If this value of x should be substituted for x in the second equation, how many unknown quantities would the resulting equation contain?

4. How, then, may an unknown quantity be eliminated from two simultaneous equations by *substitution?*

RULE. — *Find an expression for the value of one of the unknown quantities in one of the equations.*

Substitute this value for the same unknown quantity in the other equation, and solve the resulting equation.

1. Given $\left\{\begin{aligned} 3x - 2y &= 1 \\ x + 4y &= 19 \end{aligned}\right\}$ to find the values of x and y by substitution.

SOLUTION.
$$3x - 2y = 1 \qquad (1)$$
$$x + 4y = 19 \qquad (2)$$

From (1),
$$x = \frac{1 + 2y}{3} \qquad (3)$$

Substituting (3) in (2),
$$\frac{1 + 2y}{3} + 4y = 19 \qquad (4)$$

Clearing of fractions,
$$1 + 2y + 12y = 57 \qquad (5)$$

Transposing, etc.,
$$14y = 56 \qquad (6)$$
$$y = 4 \qquad (7)$$

Substituting in (3),
$$x = \frac{1 + 8}{3} \qquad (8)$$
$$x = 3 \qquad (9)$$

Solve the following by elimination by substitution:

2. $\begin{cases} 2x + y = 13 \\ x + 4y = 31 \end{cases}$

3. $\begin{cases} x - 3y = 3 \\ 2x + 4y = 16 \end{cases}$

4. $\begin{cases} 3x - 2y = 3 \\ 2x + 3y = 28 \end{cases}$

5. $\begin{cases} 4x + 4y = 76 \\ 3x + y = 39 \end{cases}$

6. $\begin{cases} 3x + y = 14 \\ x + 3y = 26 \end{cases}$

7. $\begin{cases} 4x + y = 25 \\ y - 4x = 9 \end{cases}$

8. $\begin{cases} 7x - 4y = 81 \\ 2x - y = 25 \end{cases}$

9. $\begin{cases} x + 2y = 20 \\ x - 2y = 4 \end{cases}$

10. $\begin{cases} 3x - y = 5 \\ 3y - x = 9 \end{cases}$

11. $\begin{cases} 2x - y = 18 \\ x - 2y = -6 \end{cases}$

12. $\begin{cases} 3x - 2y = 24 \\ 2x - y = 20 \end{cases}$

13. $\begin{cases} 4x - y = 42 \\ 6x + 5y = 180 \end{cases}$

14. $\begin{cases} 5x + 3y = 46 \\ 3x + 2y = 29 \end{cases}$

15. $\begin{cases} 2x - y = 10 \\ 16x - 5y = 200 \end{cases}$

16. $\begin{cases} x + \dfrac{y}{2} = 16 \\ \dfrac{x}{2} + y = 20 \end{cases}$

17. $\begin{cases} 4x - 3y = 24 \\ 3x - 2y = 19 \end{cases}$

18. $\begin{cases} \dfrac{x}{a} + \dfrac{y}{b} = 3 \\ \dfrac{x}{a} - \dfrac{y}{b} = 2 \end{cases}$

19. $\begin{cases} \dfrac{2x}{3} - \dfrac{y}{2} = 0 \\ \dfrac{x}{7} + \dfrac{y}{7} = 5 \end{cases}$

20. $\begin{cases} \dfrac{a}{x} + \dfrac{b}{y} = m \\ \dfrac{c}{x} + \dfrac{d}{y} = n \end{cases}$

Solve by any method:

21. $\begin{cases} 3x-7y=13 \\ 4x+2y=40 \end{cases}$

22. $\begin{cases} 3x-3y=-6 \\ 7y-3x=22 \end{cases}$

23. $\begin{cases} x+5y=7 \\ 4x+3y=11 \end{cases}$

24. $\begin{cases} 4x-2y=6 \\ 3x+3y=36 \end{cases}$

25. $\begin{cases} x-\dfrac{y}{2}=8 \\ \dfrac{x}{2}-y=7 \end{cases}$

26. $\begin{cases} \dfrac{x}{3}-\dfrac{y}{5}=3 \\ \dfrac{x}{5}+\dfrac{y}{2}=8 \end{cases}$

27. $\begin{cases} \dfrac{2x}{3}-\dfrac{y}{8}=2 \\ \dfrac{x}{6}-\dfrac{y}{4}=-3 \end{cases}$

28. $\begin{cases} \dfrac{x}{2}+\dfrac{y}{2}=2 \\ \dfrac{x}{4}+y=2\frac{1}{2} \end{cases}$

29. $\begin{cases} 8x-15y=19 \\ 2x+y=19 \end{cases}$

30. $\begin{cases} 4x+2y=24 \\ 3x-y=8 \end{cases}$

31. $\begin{cases} x+5y=12 \\ \dfrac{x}{2}+3y=7 \end{cases}$

32. $\begin{cases} x+y=a \\ x-y=b \end{cases}$

33. $\begin{cases} ax+by=c \\ bx-ay=d \end{cases}$

34. $\begin{cases} \dfrac{a}{x}+\dfrac{b}{y}=m \\ \dfrac{a}{x}-\dfrac{b}{y}=n \end{cases}$

35. $\begin{cases} \dfrac{x}{m}+ny=m+n \\ \dfrac{mx}{n}+\dfrac{n^3y}{m}=m^2+n^2 \end{cases}$

36. $\begin{cases} \dfrac{x}{a}+\dfrac{y}{b}=1 \\ \dfrac{x}{b}+\dfrac{y}{a}=1 \end{cases}$

139. Three unknown quantities.

1. Given $\left\{\begin{array}{r} 2x+\ \ y+3z=13 \\ 3x+2y+\ \ z=10 \\ x+3y+2z=13 \end{array}\right\}$ to find x, y, and z.

Solution.

	$2x+y+3z=13$	(1)
	$3x+2y+z=10$	(2)
	$x+3y+2z=13$	(3)
$(2)\times 3$,	$9x+6y+3z=30$	(4)
(1),	$2x+y+3z=13$	
$(4)-(1)$,	$7x+5y=17$	(5)
$(2)\times 2$,	$6x+4y+2z=20$	(6)
(3),	$x+3y+2z=13$	
$(6)-(3)$,	$5x+y=7$	(7)
$(7)\times 5$,	$25x+5y=35$	(8)
(5),	$7x+5y=17$	
$(8)-(5)$,	$18x=18$	(9)
	$x=1$	(10)
Substituting (10) in (7),	$5+y=7$	(11)
	$y=2$	(12)
Substituting (10) and (12) in (1),	$2+2+3z=13$	(13)
	$z=3$	(14)

The student will observe that by combining two of the given equations *one* of the unknown quantities is eliminated, and that by combining another pair of the given equations the same quantity is eliminated. We have thus two equations containing two unknown quantities which may be readily solved.

Find the value of each unknown quantity in the following:

2. $\left\{\begin{array}{r} x+\ \ y+\ \ z=6 \\ x+2y+\ \ z=7 \\ x+\ \ y+2z=9 \end{array}\right\}$

3. $\left\{\begin{array}{r} 2x+2y+\ \ z=15 \\ 3x+\ \ y+2z=23 \\ x-3y+2z=11 \end{array}\right\}$

4. $\begin{cases} x+2y+z=16 \\ 4x-3y-z=6 \\ 2x+2y-z=11 \end{cases}$

5. $\begin{cases} 5x-y+2z=38 \\ 2x+y-z=4 \\ x-3y+5z=44 \end{cases}$

6. $\begin{cases} x+y+z=15 \\ 2x+2y-z=6 \\ x-y+4z=37 \end{cases}$

7. $\begin{cases} x+2y-z=6 \\ 4x-y+2z=27 \\ 2x+2y+z=31 \end{cases}$

8. $\begin{cases} 3x-5y+2z=6 \\ 5x+y-3z=14 \\ 4x+2y-4z=-4 \end{cases}$

9. $\begin{cases} x-y+z=17 \\ x+4y-6z=4 \\ 3x-2y-3z=7 \end{cases}$

10. $\begin{cases} -2x+2y+z=9 \\ 5x-3y=7 \\ 4y+z=31 \end{cases}$

11. $\begin{cases} x+y=14 \\ 2y-3z=9 \\ 5x-z=5 \end{cases}$

12. $\begin{cases} 2x-y=9 \\ x-2z=3 \\ y-2z=5 \end{cases}$

13. $\begin{cases} x-12y+2z=-20 \\ 2x-y-z=7 \\ x+z=19 \end{cases}$

14. $\begin{cases} 6x-y+z=28 \\ 5x+2y-2z=12 \\ 2x+2y-z=8 \end{cases}$

15. $\begin{cases} \frac{x}{4}+\frac{y}{8}=5 \\ \frac{x}{4}+\frac{z}{10}=5 \\ \frac{y}{4}+\frac{z}{5}=8 \end{cases}$

16. $\begin{cases} \frac{x}{2}+\frac{y}{5}+z=9 \\ \frac{x}{4}+\frac{y}{2}+z=10 \\ x+y+\frac{z}{3}=19 \end{cases}$

17. $\begin{cases} x+y=9 \\ x+z=12 \\ y+z=15 \end{cases}$

18. $\begin{cases} y-x+z=-5 \\ z-y-x=-25 \\ x+y+z=35 \end{cases}$

19. $\begin{cases} 2x-3y=1 \\ 3y-4z=7 \\ 4z-5x=-32 \end{cases}$

20. $\begin{cases} \frac{1}{x}+\frac{1}{y}-\frac{1}{z}=\frac{2}{3} \\ \frac{1}{x}-\frac{1}{y}+\frac{1}{z}=\frac{1}{3} \\ \frac{1}{y}+\frac{1}{z}-\frac{1}{x}=0 \end{cases}$

22. $\begin{cases} 2x-y=6 \\ x+y-2z=-2 \\ 4x-3y+z=11 \end{cases}$

21. $\begin{cases} \frac{1}{x}+\frac{2}{y}=\frac{3}{5} \\ \frac{3}{y}-\frac{4}{z}=\frac{1}{5} \\ \frac{3}{z}-\frac{4}{x}=-\frac{1}{2} \end{cases}$

23. $\begin{cases} \frac{x}{2}-\frac{z}{15}=8 \\ \frac{z}{5}-\frac{y}{5}=1 \\ \frac{x}{4}-\frac{y}{5}=0 \end{cases}$

24. $\begin{cases} 25x-20y+15z=80 \\ 15x-25y+20z=60 \\ 20x+15y-25z=-40 \end{cases}$

25. A farmer has sheep in three pastures. The number in the first plus $\frac{1}{2}$ of the number in the second plus $\frac{1}{3}$ of the number in the third equals 70. The number in the first plus $\frac{1}{4}$ of the number in the second minus $\frac{1}{6}$ of the number in the third equals 30. The number in the second plus $\frac{1}{2}$ of the number in the first plus $\frac{1}{10}$ of the number in the third equals 61. How many sheep are there in each pasture?

26. Henry, James, and Ralph together have 50 cents. Henry's and Ralph's money amounts to 35 cents; James' and Ralph's amounts to 40 cents. How many cents has each?

27. A farmer has wheat, oats, and barley. The number of bushels of wheat and oats is 200; the number of bushels of wheat and barley is 190; and the number of bushels of oats and barley is 90. How many bushels are there of each kind of grain?

GENERAL REVIEW.

I.

140. 1. When $a=1$, $b=2$, $c=4$, $d=6$, what is the value of $\frac{2a+b^2-ab+2c}{d-c+ab}-\frac{cd-bc}{cd-b}$?

2. Find the sum of $3a(x-y)$, $4a(x-y)$, $2b(x-y)$, and $5(x-y)$.

3. Simplify $(x+5)-(x+10)-[x-(3x+25)-10]$.

4. Add $7\sqrt{x}-8x+7x^2$, $-6\sqrt{x}+4x^2-8x$, $7x^2-8\sqrt{x}$.

5. Add $ax(a-1)+(b^2-2)+y^2$, $2(b^2-2)-3y^2+3ax(a-1)$, $5y^2-6ax(a-1)-6(b^2-2)$, and subtract from the result $4ax(a-1)+(b^2-2)-7y^2$.

6. What number added to three times itself equals ab?

7. A man had property costing $4b$ dollars, $7c$ dollars, $5bc$ dollars, which he sold for \$ 600. How much did he gain?

8. From $205x^2-74y^2+89c^2$ subtract $35x^2-4y^2-c^2+15$.

9. Simplify $x^2+y^2-\{x^2-y^2-(2x^2-4y^2)-2z\}$.

10. What number subtracted from a times itself equals a^2-1?

11. Multiply $x^4+2x^3y+4x^2y^2+8xy^3+16y^4$ by $x-2y$.

12. Expand $(x+3)(x-4)(x+4)(x-3)(x-1)(x+1)$.

13. Factor $5x^2y - 10axy + 25xy^3 - 15a^2x^2y^2$.

14. Factor $x^2 + x - 156$, $x^2 - 15x + 56$, $x^2 - 3x - 70$.

15. If 7 quarts of milk cost 1 cent less than a cents, what will b quarts of milk cost?

16. With a dollars a man paid for 10 bushels of potatoes at c dollars per bushel, and received 2 dollars in change. Express this as an equation.

17. Divide $x^4 + x^2y^2 + y^4$ by $x^2 + xy + y^2$.

18. Factor $a^3 - b^6$, $125x^3 - 64y^3$, $x^4 - 81$.

19. Divide $2x^6 - 9x^4 - 8x^3 - 1$ by $x^3 + 3x^2 + 3x + 1$.

20. Find the value of x in the equation $bcx - ab = -dx - 1$.

21. Factor $4a^2 + 36a + 81$, $100x^2 - 25y^2$, $9x^2 - 42xy + 49y^2$.

22. A merchant sold 6 pounds of coffee at b cents a pound, 5 pounds at c cents a pound, and b pounds at a cents a pound. What was the average price received per pound for the coffee?

23. Find the value of x in the equation $ax - a^2 - b^2 = 2ab - bx$.

24. Factor $8x^3 + 512$, $a^2b^2 - 1$, $m^3 + 27$.

25. Solve $(x-5)(x+4) = x(x-5) + 8$.

II.

26. Write out the following squares:

$(x+2y)^2$, $(4a-3b)^2$, $(2m-5)^2$, $(3ab+1)^2$, $(5+2a)^2$, $(6x-7y)^2$, $(a^2+10)^2$, $(m^2-4n^2)^2$, $(x^2-3y)^2$, $(3a+5x)^2$.

27. Write out the following products:

$(x+7)(x-4)$, $(x+15)(x-15)$, $(x+10)(x+3)$, $(x-12)(x+9)$, $(2x+5)(2x-5)$, $(m^2+1)(m^2-1)$, $(x-8)(x-11)$, $(x+15)(x-14)$, $(3x+7)(3x-7)$.

28. Find the highest common divisor of

$$3a^5-48a,\ 2a^3b-16b, \text{ and } 5a^2c-20c.$$

29. Find the highest common divisor of

$$x^2+6x+9,\ x^2+x-6,\ 3x^2+7x-6.$$

30. Find the lowest common multiple of

$$x^2+2x-3,\ x^2-3x+2,\ x^2+x-6.$$

31. Find the lowest common multiple of

$$x^2-9,\ x^2+9x+18,\ x^2+3x-18.$$

32. Reduce $\dfrac{a^2+ab+b^2}{a^3-b^3}$ to its lowest terms.

33. Reduce $\dfrac{x^2-4y^2}{x^2-3xy-10y^2}$ to its lowest terms.

34. Simplify $\left(\dfrac{4}{x+y}+\dfrac{2}{x-y}\right)\div\dfrac{3x-y}{x+y}$.

35. Simplify $\dfrac{5}{x+3}\times\left(\dfrac{2x^2+12}{5}-\dfrac{x^2+9}{3}\right)$.

36. Simplify $\dfrac{1}{4(1+y)}-\dfrac{1}{4(y-1)}+\dfrac{2}{2(y^2-1)}$.

37. Simplify $\dfrac{x(a-x)}{a^2+2ax+x^2}\times\dfrac{x(a+x)}{a^2-2ax+x^2}$.

38. Simplify $\dfrac{a^2-x^2}{a+b}\div\dfrac{ax+x^2}{a^2-b^2}$.

39. Simplify $\dfrac{3a-4b}{7}-\dfrac{2a-b-c}{3}+\dfrac{15a-4c}{12}$.

40. A can do a piece of work in m days; B can do it in n days. Express the part that each can do in one day. Express the part that both can do in one day, and the number of days in which both can do the work.

41. Solve $\frac{3x+5}{7}+10=\frac{3x}{5}+\frac{2x+7}{3}$.

42. Solve $x(x-3)+\frac{x}{6}=x(x-5)+\frac{13}{3}$.

43. Solve $\frac{x-2}{x^2-4}=\frac{x+2}{x^2-4}-\frac{4}{x+2}$.

44. Solve $\frac{a(a-x)}{b}-\frac{b(b+x)}{a}=x$.

45. A farmer has one third as many horses as cows, and one half as many cows as sheep. If there are a animals in all, how many are there of each kind?

46. Solve $\frac{a}{x-a}-\frac{b}{x-b}=\frac{a-b}{x-c}$.

47. Solve $\frac{5}{1-5x}+\frac{4}{2x-1}=\frac{3}{3x-1}$.

48. The sum of two numbers is a, and the first divided by m equals the second divided by n. What are the numbers?

49. The mth part of a certain number plus 10 equals m times the number minus 5. What is the number?

50. If a certain number is divided successively by a, b, and c, the sum of the quotients will be 10. What is the number?

III.

51. Expand $(x-1)(x+1)(x^2+1)(x^4+1)$.

52. Factor a^8-1, a^8-b^8, and a^4-81.

53. John purchased a pair of skates for c cents, a book for m cents, and a cap for r cents less than he paid for the book. If he offered a \$2 bill in payment, what was his change?

54. Find the value of x in the equation $a(cx-b)=d(bx-c)$. What is the value of x when $a=4$, $b=3$, $c=2$, $d=1$?

55. Expand $(a+z)(a^2-az+z^2)$ and $(1-x)(1+x+x^2)$.

56. Factor x^3-27, $1-a^4$, and $16x^3+54$.

57. Solve $(x+8)(x-7)-(x-1)(x+1)=45$.

58. A boy has 1 hour in which to return a bicycle borrowed of a friend and to walk back to school. Riding at the rate of 2 blocks in 1 minute, and walking at the rate of 1 block in 2 minutes, he has just 5 minutes to spare. How many blocks from school does his friend live?

59. Divide $a^5+4ab^4-40b^5$ by $a-2b$.

60. If 4 tons of coal cost 1 dollar more than g dollars, how much will r tons cost?

61. Factor $5a^3b-10a^2b^2c+10ab^3c^2-15ab^4c^3$.

62. What is the highest common divisor of x^2+xy+y^2 and $x^4+x^2y^2+y^4$?

63. What number is $6a-9b$ less than 4 times itself?

64. Simplify $1-[x-(1-x)-\{x-4\}-(3-x)]$.

65. Factor $x^2 - 7x + 10$, $y^2 - y - 20$, and $m^2 + m - 6$.

66. Reduce $\dfrac{m^3 + 2m^2 + 4m}{m^3 - 8}$ to its lowest terms.

67. Reduce $\dfrac{8 - a^3}{4 - a^2}$ to its lowest terms.

68. Simplify $\dfrac{1}{x-1} + \dfrac{4x}{1-x^2} - \dfrac{3}{x+1} - 1$.

69. A farmer traded a dozen eggs at m cents a dozen for b pounds of tea at c cents a pound, and received 25 cents in change. Express this in an equation.

70. Simplify $\left(\dfrac{12}{a+b} + \dfrac{12}{a-b}\right) \div \dfrac{4a^2}{a^2 - 2ab + b^2}$.

71. Simplify $\dfrac{x^3 - 3x + 2}{x - 1} \times \left(\dfrac{2}{x+2} - \dfrac{1}{1-x}\right)$.

72. Simplify $\left(\dfrac{a^6 - b^6}{a^4 - b^4} \div \dfrac{a^3 - b^3}{a^2 - b^2}\right) - (a + b)$.

73. A can do a piece of work in 12 days, and B can do it in a days. What part of the work can each do in 1 day? Express the part of the work both can do in 1 day. In how many days can both together do the work?

74. The sum of two numbers is 10, and the first divided by 2 is equal to the second divided by 3. What are the numbers?

75. A man's farm containing 440 acres is divided into four fields. If the first is $\frac{1}{2}$ as large as the second, $\frac{1}{3}$ as large as the third, and $\frac{3}{4}$ as large as the fourth, how many acres are there in each field?

IV.

76. Expand the following products into polynomials:

$(x+6)(x+3)$, $(x-10)(x+11)$, $(x+25)(x-1)$, $(x^2+1)(x^2-2)$, $(3m+5)(3m-2)$, $(2x-1)(2x-6)$, $(x-2a)(x-2a)$, $(xy+mn)(xy+mn)$, $(1-a^2x^2)(1+a^2x^2)$.

77. Solve $\frac{1}{x}=\frac{1}{3}+\frac{1}{4}-\frac{1}{12}$.

78. Solve $\frac{x}{m+x}-\frac{3x}{m-x}=4-\frac{6m^2}{m^2-x^2}$.

79. Solve $\frac{1}{x}=\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$.

80. What number divided by 3 is 4 less than the number?

81. What number diminished by its nth part is equal to a?

82. Solve $\frac{x+3}{5}-x(x-8)=(1-x)(x-10)-9$.

83. Solve $\frac{x-a}{x+a}-\frac{x+4a}{x-a}=\frac{-2a^2}{a^2-x^2}$.

84. Solve $\frac{3}{x+1}-\frac{2}{x-9}=\frac{1}{x-4}$.

85. Solve $\frac{a}{x}-\frac{a-b}{x+1}-\frac{b-c}{x-1}=\frac{c}{x}$.

86. Find the highest common divisor of $3a^3b^3-27ab$, $2a^2b^2+12ab+18$, and $a^3b^2+a^2b-6a$.

87. A man who had a dollars bought 5 gallons of molasses at m cents a gallon. How many pounds of sugar at c cents a pound can he buy with the remainder of his money?

88. Reduce $\frac{x^4y - xy^4}{x^3y - xy^3}$ to its lowest terms.

89. Reduce $\frac{(x+1)^2(x-1)^2}{x^4-1}$ to its lowest terms.

90. Simplify $\frac{m-n}{m^2-mn+n^2} - \frac{1}{m+n}$.

91. Simplify $\frac{1}{a-x} - \frac{1}{a+x} - \frac{1}{a^2-x^2}$.

92. Simplify $\frac{a}{7+a} + \frac{a+7}{7-a} + \frac{2a^2}{a^2-49}$.

Solve:

93. $\frac{x+2}{x-2} + \frac{4x+2}{3x-6} = 6.$

94. $\frac{5x+19}{2x+4} - \frac{x+14}{5x+10} = \frac{25+2x}{3x+6}.$

95. $\frac{7x+1}{9x-3} + \frac{13x+4}{6x-2} = \frac{100}{15x-5}.$

96. $\frac{1}{1+x} - \frac{2}{1-x} = \frac{3}{5+x}.$

97. $\frac{2}{6x-9} - \frac{1}{4x-4} = \frac{1}{12x}.$

98. $\frac{1}{x+1} - \frac{2}{x+2} + \frac{1}{x+4} = 0.$

99. $\frac{b}{a+b} = \frac{(a+b)^2 - a(a+b)}{x}.$

100. $\frac{x-2c}{ax-4a^2} = \frac{2}{c}.$

101. $\frac{a}{bx} + \frac{1}{a} = \frac{b}{ax} + \frac{1}{b}.$

102. What number added to twice itself equals $a+b$?

103. How much less than 1 is $a - a^2$?

V.

104. Find the highest common divisor of a^3+x^3, $(a+x)^3$, and a^2-x^2.

105. Find the highest common divisor of $9x^2-1$, $9x+3$, and $(3x+1)^2$.

106. Find the highest common divisor of x^2-2x-3, x^2+4x+3, and x^2-5x-6.

107. Find the lowest common multiple of $3(x^2+xy)$, $5(xy-y^2)$, and $6(x^2-y^2)$.

108. Find the lowest common multiple of $x^2+11x+30$, $x^2+12x+35$, and $x^2+13x+42$.

109. Find the lowest common multiple of $12xy(x^2-y^2)$, $2x(x+y)^2$, and $3y(x-y)^2$.

110. Reduce $\dfrac{3ab(a^2-b^2)}{4(a^2b-ab^2)^2}$ to its lowest terms.

111. Reduce $\dfrac{a^3+1}{a^3+3a^2+3a+1}$ to its lowest terms.

Change to equivalent fractions having their lowest common denominator:

112. $\dfrac{8x+2}{x-2}$, $\dfrac{2x-1}{3x-6}$, and $\dfrac{3x+2}{5x-10}$.

113. $\dfrac{x-3}{x^2-4}$, $\dfrac{x-2}{x^2+x-6}$, and $\dfrac{x^2+4}{x^2+6x+9}$.

114. $\dfrac{x}{x+y}$, $\dfrac{xy}{x^2-y^2}$, and $\dfrac{y^2}{x^2+y^2}$.

115. $\dfrac{3}{1+x}$, $\dfrac{5}{4+4x}$, and $\dfrac{2x}{1-x^2}$.

Simplify:

116. $\dfrac{a}{a-x}-\dfrac{x}{a+2x}-\dfrac{a^2+x^2}{(a-x)(a+2x)}.$

117. $\dfrac{x+y}{y}-\dfrac{2x}{x+y}+\dfrac{x^2y-x^3}{y(x^2-y^2)}.$

Solve:

118. $\dfrac{x-3}{4(x-1)}=\dfrac{x-5}{6(x-1)}+\dfrac{1}{9}.$

119. $\dfrac{7}{x+1}=\dfrac{6x}{x-1}-\dfrac{3(1+2x^2)}{x^2-1}.$

120. $\dfrac{17}{5(x+3)}+\dfrac{4}{5}=\dfrac{21+2x}{3(x+3)}-2.$

121. $\dfrac{3x-1}{2x-1}-\dfrac{4x-2}{3x-2}=\dfrac{1}{6}.$

122. $\dfrac{2}{2x-3}=\dfrac{6}{3x+2}-\dfrac{1}{x-2}.$

123. $am-b+\dfrac{x}{m}=\dfrac{ax}{b}.$

124. $\dfrac{3ax-2b}{3b}-\dfrac{ax-a}{2b}=\dfrac{ax}{b}-\dfrac{2}{3}.$

125. $\dfrac{ab}{x}=bc+d+\dfrac{1}{x}.$

126. $\dfrac{2}{ac}+\dfrac{2}{ab}+\dfrac{4}{bc}=2ax+bx+cx.$

127. $\dfrac{1}{2}\left(x-\dfrac{a}{3}\right)+\dfrac{1}{4}\left(x-\dfrac{a}{5}\right)=\dfrac{1}{3}\left(x-\dfrac{a}{4}\right).$

128. $\dfrac{x^2-x+1}{x-1}-\dfrac{x^2+x+1}{x+1}=\dfrac{2x+1}{x^2-1}.$

VI.

Solve the following equations:

129. $\begin{cases} 4x + 3y = 3 \\ 10x - 9y = 2 \end{cases}$

130. $\begin{cases} x + \dfrac{y+1}{4} = 6 \\ \dfrac{x+1}{4} + y = 15 \end{cases}$

131. $\begin{cases} \dfrac{x-2}{5} - \dfrac{10-x}{3} = \dfrac{y-10}{4} \\ \dfrac{2y+4}{3} - \dfrac{2x+y}{8} = \dfrac{x+13}{4} \end{cases}$

132. $\begin{cases} \dfrac{1}{x} + \dfrac{3}{y} = a \\ \dfrac{5}{x} + \dfrac{2}{y} = b \end{cases}$

133. $\begin{cases} 4x + 12y = 5 \\ 6x - 3z = 2 \\ 16x - 9z = 1 \end{cases}$

134. $\begin{cases} \dfrac{x}{2} + \dfrac{y}{3} + \dfrac{z}{4} = 62 \\ \dfrac{x}{4} + \dfrac{y}{5} - \dfrac{z}{12} = 8 \\ \dfrac{x}{3} - \dfrac{y}{4} + \dfrac{z}{5} = 17 \end{cases}$

135. $\begin{cases} x + y = a \\ x + z = b \\ y + z = c \end{cases}$

136. $3x + 2y = 8(y - x) + 2 = 26.$

137. $\begin{cases} 5x - y = 11 \\ x + 7y = 31 \end{cases}$

138. $\begin{cases} 5x - y = 27 \\ x + 3y = 15 \end{cases}$

139. $\begin{cases} 2x + 7y = 6a \\ 7x + 2y = 3a \end{cases}$

140. $\begin{cases} 3v - 4x = 2 \\ 5v + 2y = 20 \\ 5x + 3y = 20 \end{cases}$

141. $\begin{cases} x = a + y - b \\ y = b + z - c \\ z = c - x + a \end{cases}$

Solve by factoring:

142. $x^2-16x+60=0.$

143. $x^2+8x+12=0.$

144. $x^2-4x-45=0.$

145. $x^2-x-20=0.$

146. $x^2-x-72=0.$

147. $x^2+x-42=0.$

148. $x-130=2-x^2.$

149. $12x+108=x^2.$

150. $c^2-6x=9+x^2.$

VII.

151. One half of Tom's money equals $\frac{4}{7}$ of John's, and Tom has $12 more than John. How much money has each?

152. At a certain election there were two candidates; the successful candidate had a majority of 60, which was $\frac{1}{20}$ of all the votes cast. How many votes did the defeated candidate receive?

153. A lad spent on July 4th $\frac{1}{2}$ of his money and 6 cents more for firecrackers, and $\frac{1}{3}$ of his money and 4 cents more for torpedoes. If that was all of his money, how much had he?

154. A man bought a number of sheep for $225; 10 of them having died, he sold $\frac{4}{5}$ of the remainder at cost for $150. How many sheep did he buy?

155. John had $\frac{4}{5}$ as much money as his brother, but after each had contributed 25 cents to buy a hat John had only $\frac{3}{4}$ as much money as his brother. How much had each?

156. A man divided $6100 among his three grown sons in such a way that the youngest son received $\frac{1}{5}$ less than the second son, and the second son $\frac{1}{5}$ less than the oldest son. What was the amount given to each son?

157. If to the numerator of a certain fraction 1 is added, the value of the fraction becomes 1; but if 3 is subtracted from the denominator, the value becomes 2. What is the fraction?

Suggestion. Let $\frac{x}{y}$ represent the fraction.

158. A man bought 20 bushels of wheat and 15 bushels of corn for $36, and 15 bushels of wheat and 25 bushels of corn, at the same rate, for $32.50. How much did he pay per bushel for each?

159. If $\frac{2}{3}$ of the value of a carriage is equal to $\frac{3}{4}$ of the value of a horse, and the value of the carriage is $20 more than the value of the horse, what is the value of each?

160. A merchant, after selling from a cask of vinegar 15 gallons more than $\frac{1}{7}$ of the whole, found that he had left just four times as much as he had sold. How many gallons did the cask contain at first?

161. A and B together have 100 acres more than a section of land. If $\frac{2}{7}$ of A's farm is as large as $\frac{3}{8}$ of B's farm, how many acres are there in each farm?

162. The sum of two numbers is 35, and one of them is $\frac{1}{3}$ larger than the other. What are the numbers?

163. A and B suffer equal losses by fire. B, who was worth $4000 more than A, loses $\frac{3}{5}$ of his property, while A has left only $\frac{1}{4}$ of his property. What was A's property before the fire? How much did each man lose?

164. A fruit vender buys apples at the rate of 5 for 3 cents, and sells them at the rate of 3 for 5 cents. How many must he sell to pay $4.00 for a license?

165. A and B own flocks of sheep. If A sells to B 10 sheep, they will each have an equal number; but if B sells to A 10 sheep, A will have three times as many as B. How many sheep has each?

166. A grocer will sell 1 pound of tea, 2 pounds of coffee, and 1 pound of sugar for $1.00; or he will sell 2 pounds of tea and 6 pounds of sugar for $1.00; or he will sell 3 pounds of coffee and 2 pounds of sugar for $1.00. What is the price per pound of each?

167. How far may a person ride in a stage, going at the rate of 8 miles an hour, if he is gone 11 hours, and walks back at the rate of 3 miles an hour?

168. I have two purses, one containing gold coins, and the other silver coins. The money in both purses equals in value twice the value of the gold coins; and $\frac{1}{2}$ the value of the silver coins, increased by $\frac{1}{2}$ the value of the gold coins, equals $150. How much does each purse contain?

169. A man bought a horse, a cow, and a sheep for a certain sum. The horse and the sheep cost 5 times as much as the cow, and the sheep and the cow cost $\frac{2}{7}$ as much as the horse. How much did each cost, if the cow cost $30?

170. The sum of two numbers is 38, and $\frac{1}{3}$ of one is 2 more than $\frac{1}{5}$ of the other. What are the numbers?

171. A ball, a bat, and a suit cost $5.25. If the suit cost 2 times as much as the bat, and the bat cost 50 cents more than half as much as the ball, what was the cost of each?

172. A woman sold some butter for 25 cents a pound. Had she received 5 cents more for 1 pound less, she would have received 2 cents more per pound. How many pounds did she sell?

173. A and B have the same income. A saves $\frac{1}{6}$ of his, but B, by spending $100 each year more than A, at the end of 5 years finds himself $240 in debt. What is the income of each?

174. A farmer had his sheep in 3 fields. $\frac{2}{3}$ of the number in the first field was equal to $\frac{3}{4}$ of the number in the second field, and $\frac{2}{3}$ of the number in the second field was $\frac{3}{4}$ of the number in the third field. If the entire number was 434, how many were there in each field?

175. A man left one half of his property to his wife, one sixth to his daughter, one twelfth to his brother, and the rest, which was $900, to charitable institutions. How much was his property worth?

176. A and B together have $1340. A buys horses at $60 a head, and B buys five times as many sheep at $3.25 a head: If each man now has $60 left, how many horses did A buy?

177. The sum of two numbers is a, and their difference is b times the smaller. What are the numbers? What are the numbers when a is 24 and b is 2?

178. A huckster sold 25 bushels of apples for $18.50, some at 20 cents a peck, the rest at 15 cents a peck. How many bushels of each kind did he sell?

179. A milkman who had been buying milk for 10 cents a gallon, and selling it at the rate of 18 quarts for $1, reduced the selling price to $1 for 22 quarts. If by that means he increased his daily sales 300 quarts, and his daily profits $2.50, how many quarts did he sell a day at 18 quarts for $1?

180. A square pasture is made 15 rods narrower and 20 rods longer by moving the fences on two sides. If there is the same area of pasture as before, what are the dimensions of the two fields?

VIII.

Solve:

181. $\begin{cases} 2x-3y+z=1 \\ 4x+2y-3z=13 \\ 3x-5y+4z=3 \end{cases}$

182. $\begin{cases} \dfrac{x}{2}+\dfrac{y}{4}+\dfrac{z}{6}=3 \\ \dfrac{x}{5}+y+\dfrac{z}{10}=5 \\ \dfrac{x}{4}+y-\dfrac{z}{12}=4 \end{cases}$

183. $\begin{cases} \dfrac{x-a}{b}=\dfrac{b-y}{a} \\ \dfrac{x+y-b}{a}=\dfrac{a+y-x}{b} \end{cases}$

184. $\begin{cases} ax+by=c^2 \\ \dfrac{a}{b+y}=\dfrac{b}{a+x} \end{cases}$

185. $\begin{cases} \dfrac{x-a}{y-b}=c \\ a(x-a)+b(y-b)=-abc \end{cases}$

186. $\begin{cases} \dfrac{x}{a+b}+\dfrac{y}{a-b}=2a \\ \dfrac{x-y}{4ab}=1 \end{cases}$

187. $\begin{cases} \dfrac{x}{3}+\dfrac{x+2}{4}-\dfrac{y+4}{5}=\dfrac{y-2}{2} \\ \dfrac{y+5}{3}+\dfrac{2z+6}{5}=z+1 \\ \dfrac{x+6}{6}-\dfrac{z-x}{2}-\dfrac{x}{6}+\dfrac{z}{4}=0 \end{cases}$

188. $\begin{cases} \dfrac{3y-2z}{8}=\dfrac{y-4}{5} \\ \dfrac{3y+2z}{7}=\dfrac{y+2z}{4} \end{cases}$

189. $\begin{cases} \dfrac{x+y-2}{7}=\dfrac{x-y+2}{5} \\ \dfrac{x-y-10}{4}=\dfrac{x-3y+3}{3} \end{cases}$

190. A person being asked the time of day, replied that it was past noon, and that $\frac{1}{2}$ of the time past noon was equal to $\frac{1}{4}$ of the time to midnight. What was the time?

191. A yacht, whose rate of sailing in still water is 12 miles an hour, sails down a river whose current is 4 miles an hour. How far may it go, if it is to be gone 15 hours?

192. Three men engage to husk a field of corn. The first can do it in 10 days, the second in 12 days, and the third in 15 days. In what time can they do it together?

193. Fifteen persons agree to purchase a tract of land, but three of the company withdrawing, the investment of each of the others is increased $150. What is the cost of the land?

194. C and D have the same income. C saves $\frac{1}{12}$ of his, but D, by spending $65 more each year than C, at the end of 6 years finds himself $60 in debt. How much does each spend yearly?

195. I sold a bureau to A for $\frac{1}{4}$ more than it cost me. He sold it for $6, which was $\frac{2}{5}$ less than it cost him. How much did the bureau cost me?

196. A man agreed to work 16 days for $24 and his board, but he was to pay $1 a day for his board every day he was idle. If he received $14, how many days did he work?

197. A man can saw 2 cords of wood per day, or he can split 3 cords of wood after it is sawed. How much must he saw that he may be occupied the rest of the day in splitting it?

198. A carriage maker sold 2 carriages for $300 each. Did he gain or lose by the sale, if on one he gained 25 per cent, and on the other he lost 25 per cent?

199. A steamboat makes a trip from P to H and return in 10 hours, stopping 40 minutes at H. If it goes 6 miles an hour upstream and 10 miles an hour downstream, what is the distance between P and H?

200. A man has two horses, and a saddle worth \$10. The value of the saddle and the first horse is double that of the second horse, but the value of the saddle and the second horse lacks \$13 of being equal to the value of the first horse. What is the value of each horse?

201. How many quantities each equal to $a^2 - 2a + 1$ must be added together to produce $5a^6 - 6a^5 + 1$?

202. There is a cistern into which water is admitted by three faucets, two of which are of the same size. When they are all open the cistern will be filled in 6 hours, but if one of the equal faucets is closed the other two will require 8 hours and 20 minutes to fill it. In what time can each faucet fill the cistern?

203. A farmer purchased 100 acres of land for \$2450. For a part of it he paid \$20 an acre and for the rest \$30 an acre. How many acres were there in each part?

204. A said to B, "Give me \$100, and I shall have as much money as you have left." B said to A, "Give me \$100, and I shall have three times as much money as you have left." How much money had each?

205. The head of a fish is 8 inches long. The tail is as long as the head and $\frac{1}{2}$ of the body, and the body is as long as the head and tail. What is the length of the fish?

206. A tree is broken into three pieces. The part standing is 8 feet long. The top piece is as long as the part standing and $\frac{1}{4}$ of the middle piece, and the middle piece is twice as long as the other pieces. How high was the tree?

QUESTIONS FOR REVIEW.

141. How does the algebraic solution of a problem differ from the arithmetical solution?

What are the letters used in algebra commonly called?

Define unknown numbers or quantities.

What letters are used to represent unknown quantities?

What is an equation?

Give the sign of equality; the sign of deduction.

What is an algebraic expression? Illustrate.

How do the uses of signs in algebra differ from their uses in arithmetic?

What is a power? Give an illustration.

What is an exponent? Give an illustration.

How are powers named? What, also, is the *second* power called? What, the *third*?

What is a coefficient? When no coefficient is expressed, what is the coefficient? Give an illustration.

How should quantities inclosed in parentheses be treated?

What is an algebraic term?

Distinguish between positive and negative terms. Illustrate each.

Distinguish between similar and dissimilar terms. Illustrate each.

What is a monomial? A polynomial? A binomial? A trinomial? Give an illustration of each.

Explain what is meant by using the signs + and − as signs of opposition.

When positive quantities are added, what is the sign of the sum?

What is the sign of the sum when negative quantities are added?

How is the sign of the result determined when both positive and negative quantities are added?

What kind of quantities can be united by addition into one term?

How may dissimilar quantities be treated in addition?

What are the two cases in addition? Give the rule.

Define known numbers or quantities. What letters of the alphabet represent them?

Instead of subtracting a positive quantity, what may be done to secure the same result?

Instead of subtracting a negative quantity, what may be done to secure the same result?

Give the three principles in subtraction.

Give the cases in subtraction.

When is it necessary in subtraction to inclose the coefficient of the answer in parentheses?

Give the signs of aggregation. What do they show?

How may the subtrahend sometimes be expressed?

When the subtrahend is inclosed in parentheses and preceded by the sign minus, what must be done when the subtraction is performed?

Give the two principles relating to parentheses, or other signs of aggregation.

How does the sign *plus* before parentheses affect the quantity inclosed?

When a quantity is changed from one member of an equation to another, what change must be made in its sign?

What are the members of an equation? What is the first member? The second member? Illustrate.

What is an axiom? Give the six axioms.

What is transposition? Give the principle relating to transposition. Give the rule.

How may the value of the unknown quantity obtained by solving an equation be verified? Illustrate the process by an appropriate example.

When a *positive* quantity is multiplied by a *positive* quantity, what is the sign of the product?

When a *negative* quantity is multiplied by a *positive* quantity, or a *positive* quantity by a *negative* quantity, what is the sign of the product?

When a *negative* quantity is multiplied by a *negative* quantity, what is the sign of the product?

Name the four ways of indicating multiplication.

Give the three principles of multiplication. Give the cases in multiplication. Give the rules.

What is the principle relating to the square of the sum of two quantities? What is the square of $x + y$?

What is the principle relating to the square of the difference of two quantities? What is the square of $x - y$?

What is the principle relating to the product of the sum and difference of two quantities? What is the product of $(x + y)(x - y)$?

What is the principle relating to the product of two binomials which have a common term? What is the product of $(x + 3)(x + 2)$?

What are simultaneous equations? Illustrate.

Define elimination. Give the principle relating to elimination by addition or subtraction. Give the rule.

Illustrate the method of elimination by addition or subtraction by the solution of a problem.

What is the sign of the quotient when a *positive* quantity is divided by a *positive* quantity?

What is the sign of the quotient when a *negative* quantity is divided by a *negative* quantity?

What is the sign of the quotient when a *positive* quantity is divided by a *negative* quantity, or a *negative* quantity is divided by a *positive* quantity?

How is the exponent of a quantity in the quotient found?

Give the principles relating to division. Give the cases in division. Give the rule under each case.

What is a factor? What is factoring? Give the rule for factoring a polynomial all of whose terms have a common factor. How may a polynomial be factored when only some of its terms have a common factor?

What is meant by "square root"? Give the rule for separating a trinomial into two equal factors. When can a binomial be resolved into two binomial factors? Give the rule for so factoring a binomial.

What is a quadratic trinomial? Illustrate the method of factoring a quadratic trinomial.

What quantity will divide the sum of two cubes? How, then, may the sum of two cubes be factored? Give the factors of $x^3 + y^3$. What quantity will divide the difference of two cubes? How, then, may the difference of two cubes be factored? Give the factors of $x^3 - y^3$.

Explain the solution of the four kinds of equations by factoring.

What is a common divisor? Illustrate the use of the common divisor by giving two or more quantities and a common divisor or factor of them. What is the highest common divisor or factor of several quantities? Give the principle relating to the highest common divisor.

What is a multiple of a quantity? What is a common multiple of several quantities? What is the lowest common multiple of several quantities? Give the principle relating to the lowest common multiple. Find the lowest common multiple of two or more quantities which are monomials, and of two or more which are polynomials.

Define a fraction; an entire quantity; a mixed quantity. Illustrate each. What is meant by the sign of a fraction? Give the principle relating to reduction of fractions. When is a fraction in its lowest terms?

Name the five cases under reduction of fractions, and illustrate by solving examples. Define and illustrate similar and dissimilar fractions. What is meant by lowest common denominator? Illustrate. Give the principles relating to the lowest common denominator.

What effect has it upon the equality of the members of an equation to multiply both members by the same quantity?

How may an equation with fractions be changed into one without fractions? What is meant by clearing an equation of fractions? Give the principle and the rule.

If a fraction has the minus sign before it, what must be done when the denominator is removed? If a fraction is multiplied by its denominator, what is the effect? What must be done to dissimilar fractions before they can be added or subtracted? Give the principles.

In what two ways may a fraction be multiplied, and in what two ways may it be divided by an entire quantity? How should entire and mixed quantities be changed before multiplying? How may an entire quantity be changed to a fractional form?

Solve an example in multiplication of fractions and one in division of fractions, making use of factoring and cancellation. Explain the reason for inverting the divisor in division of fractions.

Define and illustrate the term simple equation.

How may an unknown quantity be eliminated from two simple simultaneous equations by comparison? By substitution? Give the rules. Illustrate by solving examples.

Describe the method of solving, and solve a set of simple simultaneous equations containing three unknown quantities.